Name (IN CAPITAL LETTERS!): ....

CID: ....

Question 4. In each case, consider the value of the integral (if it exists), select the correct answer from the alternatives and write **a, b, c** or **d** in the box below. No marks will be given for working and there is no penalty for wrong answers, but anything unclear will score zero. "**DE**" below stands for "Doesn't exist." while "**SE**" means "Something Else," i.e. none of the other 3 answers is correct.

(1) $\displaystyle\int_1^\infty \left[\sin x + \frac{1}{x^2}\right] dx$ (a) **DE** (b) $1+\cos 1$ (c) $1-\cos 1$ (d) **SE**.

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(2) $\displaystyle\int_{-2}^{-1} \frac{dx}{x}$ (a) **DE** (b) $-\log 2$ (c) $\log 2 + n\pi i$ (d) **SE**.

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(3) $\displaystyle\int_1^3 \frac{dx}{\sqrt{x^2-1}}$ (a) **DE** (b) $\frac{2}{3}\sqrt{2}$ (c) $\log(3+2\sqrt{2})$ (d) **SE**.

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(4) $\displaystyle\int_{-1}^{1} \frac{\sin^3 x}{x^2+\cosh x - 1}\, dx$ (a) **DE** (b) $\frac{2}{3}\pi+\log(1+\cosh^{-1} 1)$ (c) 0 (d) **SE**.

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(5) $\displaystyle\int_1^e \frac{dx}{x\log x}$ (a) **DE** (b) 1 (c) e^{-1} (d) **SE**.

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(6) $\displaystyle\int f(x)f'''(x)\, dx$ (a) $\frac{1}{2}f^2f''+C$ (b) $ff''-\frac{1}{2}(f')^2+C$ (c) $\frac{1}{2}(f'')^2+C$ (d) **SE**.

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(7) $\displaystyle\int_0^y \frac{dx}{x+\sec^2 x}$ (a) **DE** (b) $\log(y+\sec^2 y)$ (c) $\log y+\tan y$ (d) **SE**.

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(8) $\displaystyle\int_0^x \exp(e^t+t)\, dt$ (a) $\exp(e^x-1)$ (b) $(e^x+x)\exp(e^x+x)$ (c) $\exp(e^x)-e$ (d) **SE**.

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(9) $\displaystyle\int_3^\pi x^{20}(\cos x - 1)\, dx$ (a) about 10^9 (b) about 10^2 (c) between 0 and 1 (d) **SE**.

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(10) $\displaystyle\int_0^1 \frac{\tan x}{\sin x \cos x}\, dx$ (a) **DE** (b) $\frac{1}{4}\pi$ (c) $\tan(1)$ (d) **SE**.

	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**	**(9)**	**(10)**
ANSWER:	a	b	c	c	a	b	d	c	d	c

Explanatory comments:

1. $\sin x$ cannot be integrated up to infinity – the integral does not converge.

2. The integral of $1/x$ is technically $\log|x|$, as it has a real value even when $x < 0$. To see this, substitute $y = -x$.

3. Substitute $x = \cosh t$, to find the integral is $\cosh^{-1} 3 - \cosh^{-1} 1 = \cosh^{-1}(3)$. This can be written as a logarithm, as shown earlier in the course, $\log(3 + \sqrt{3^2 - 1})$. Note the singularity at $x = 1$ is a square root, which is integrable.

4. The denominator is never zero, so integral exists. The indefinite integral cannot be evaluated, but the integrand is **odd**, and so $\int_{-1}^{1} = 0$, by symmetry.

5. This integrates to $\log|\log x|$. As $x \to 1$ this blows up, so integral doesn't exist.

6. Check by differentiation: $(ff'' - \frac{1}{2}(f')^2)' = ff''' + f'f'' - f'f'' = ff'''$.

7. Denominator is only zero for $x < 0$. Question ought to specify $y > 0$. Then integral exists, but both answers are rubbish.

8. Confusing, but $(\exp(e^x))' = e^x \exp(e^x)$ and so (c) is correct.

9. Integral exists, and the integrand is negative, so the answer must be negative. Hence (d). We could estimate it using the mean value theorem for integrals: It is $(\pi - 3)\xi^{20}(\cos\xi - 1)$ for some ξ in $3 < \xi < \pi$ Thus the answer is about $-2(\pi - 3)3^{20}$ which looks like -10^9.

10. Integrand simplifies to $\sec^2 x$, which integrates to $\tan x$.