M1M1: Problem Sheet 5: Curve sketching

1. Sketch the graph of the function

$$y = \frac{x+3}{2x+1},$$

carefully indicating any special features on your graph.

2. Sketch graphs of the functions:

$$(a)\ \ x^2 - 6x + 8; \quad (b)\ \ x\ \exp(-x^2); \quad (c)\ \ \frac{\cos x + \sin x}{\sqrt{2}}.$$

3. If

$$y(x) = \frac{x^2 + 2x + 5}{x+1},$$

show that

$$y(x) = x + 1 + \frac{4}{x+1}.$$

Hence sketch the graph of the function $y = y(x)$, carefully identifying any asymptotes, stationary points or points of inflexion.

4. Sketch the graph of the function

$$y = \frac{x(x-2)}{x-3},$$

carefully indicating any special features on your graph.

5. Sketch a graph of the function

$$y^2 = x(x^2 - 1),$$

carefully indicating any special features on your graph.

6. A curve is given, in polar coordinates, by the formula

$$r = \frac{2}{3 + \cos\theta}.$$

Sketch this curve.

7. A curve is given in parametric form as

$$x(t) = a\cos^3 t; \quad y(t) = b\sin^3 t,$$

where a and b are positive constants. Sketch the curve carefully, noting how dy/dx behaves at the maximum and minimum values of y.

8. Consider the function

$$f(x) = \frac{x^3 - 1}{x^3 + 1}.$$

(a) Put $f(x)$ in partial fraction form;

(b) Find and classify all the stationary points of $f(x)$;

(c) Find all the points of inflexion;

(d) Sketch the graph of $f(x)$ carefully indicating all the important features of the graph on your sketch (including the stationary points and points of inflexion).

9. Plot the curve given parametrically by

$$x = \cos t, \qquad y = \frac{|\sin t|}{t} \qquad \text{for } -2\pi \le t \le 2\pi.$$

[Hint: Consider carefully what happens as t passes through zero.)

10. The function $h(x)$ is defined for $x > 1/2$ by

$$h(x) = \sqrt{x + \sqrt{2x-1}} - \sqrt{x - \sqrt{2x-1}}$$

Sketch the curve $y = h(x)$ for $x > 1/2$.

[Important hint: before you start, try to simplify $h(x)$, by considering h^2. Recall that the square roots always take positive values.]