Questions A

1. Thomas is happy only if Mestel is unhappy. Suppose Mestel is happy. Is Thomas

   (a) Definitely happy, (b) Possibly happy or unhappy, or (c) Definitely unhappy ?

2. If Mestel is happy, Thomas is unhappy with probability 0.9 and if Mestel is unhappy, Thomas is happy. Thomas is happy half of the time and Mestel is happy a quarter of the time. Given that Thomas is happy what is the probability that Mestel is also happy? (give your answer as a fraction in its simplest form).

3. The maths department at the University of Impioxcam consists of k very unfriendly lecturers, where $k \geq 4$. Each lecturer is friends with exactly 2 other lecturers. For any 2 lecturers who aren't friends, there is exactly 1 other lecturer who is friends with both of them. The total number k is lecturers is

   (a) 4 (b) 5 (c) 6 (d) could be any integer ≥ 4

4. How many (complex) solutions are there to the equation

$$|z| = \Re e[z] + \Im m[z],$$

   where $\Re e$ and $\Im m$ denote real and imaginary parts?

   (a) As many times as Martin Liebeck has climbed Mount Everest.

   (b) As many times as Richard Thomas gave a M1F lecture on a Thursday this term.

   (c) More times than Emma McCoy has had hot dinners.

   (d) None of these.

5. The total score, X, in your weekly M1S test has the following pmf:

$$f_X(x) = \frac{c}{2^{x+1}}, \quad x = 0, \ldots, 10.$$

   What is the value of c (as a fraction in its slimplest form)?

6. What are the roots of $1 - z + z^2 - z^3 + \underset{\dots}{} - z^{2n-1}$?

7. True or false: the sum of the eigenvalues of the matrix $\begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}$ does not depend on a or b.

8. The function $f(x) = x\sin(1/x)$ for $x \neq 0$, $f(0) = 0$ is

   (a) an odd function and is continuous for all x.

   (b) a periodic function with period 0.

   (c) discontinuous at $x = 0$ and $f(x) \to 1$ as $x \to \infty$

   (d) an even function with at least three roots.

Questions B

1. A box has three drawers; one contains two gold coins, once contains two silver coins and one contains one gold and one silver coin. Assume that a drawer is selected at random and a randomly selected coin from that drawer turns out to be gold. What is the probability that the chosen drawer is the one that contains two gold coins.

2. In the specially designed Maths department snack machine you can use positive and negative numbers of 12p and 20p coins. It accepts no other coins, and charges a positive amount for all items. What's the least it can charge for any snack ?

3. In the specially designed Maths department snack machine you can use positive and negative numbers of 12p and 20p coins. It accepts no other coins, and charges a positive amount for all items. You select coins from an infinite supply in which the probability of selecting the positive 12p is twice as likely as selecting any other coin. You choose three coins at random and put them in the machine. What is the expected total value of the 3 coins?

4. I asked my colleagues what their favourite function was.

   Thomas said "You nit, Mestel" (which I misheard as "unit"), $T(x) = 1$

   Mccoy said "I'm totally normal," $M(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$

   Liebeck said "Sorry, continuous variables mean nothing to me." $L(x) = 0$. Assuming for all $x > 0$, we have $T(x) > M(x) > L(x)$, what can we say as $x \to \infty$?

   (a) $T > \lim_{x\to\infty} M > L$

   (b) $M(x)$ has no limit as $x \to \infty$

   (c) $T = M = L$ as $x \to \infty$.

   (d) None of these.

5. McCoy and Thomas are both giving the same course, with lectures timetabled at the same times, on "Fundaments of Analistics" to a class of 7 students. If a student is in Thomas's lecture one day, she (or he) is so baffled that the next day she goes to McCoy's lecture. If a student is in McCoy's lecture one day, she is inspired but out of sympathy she throws a die, and if she gets a 6 goes to Thomas next day, otherwise to McCoy again. At the 277^{th} lecture of the course, Thomas can expect an attendance of

   (a) 0 (b) 1 (c) 2 (d) 3 (e) 7

6. True or False: Any function which is differentiable everywhere is integrable over any finite interval.

7. What condition on $f : X \to Y$ is necessary for there to exist $g : Y \to X$ such that $g \circ f = \mathrm{id}_X$?

   (a) Injective (b) Surjective (c) Bijective (d) More than one of these (e) None of these

   Is your condition sufficient ?

8. The conic $x_1x_2 = 1$ is (a) an ellipse (b) a parabola (c) a hyperbola (d) degenerate

Questions C

1. Let C be the set of points in $\mathbb{R}^2$ on the unit circle $x^2 + y^2 = 1$. Is it possible to define addition and scalar multiplication in such a way as to make C a vector space over $\mathbb{R}$?

   (a) Yes (b) No

2. The infinite series
$$S = \sum_{n=1}^{\infty} \frac{x^n}{n}$$
   (a) $S = -\log(1-x)$ provided $|x|$ is small enough
   (b) has infinite radius of convergence
   (c) describes a function whose n'th derivative $f^{(n)}(0) = 1/n$
   (d) none of these

3. How any functions are there from $S := \{M1F, M1S, M1GLA, M1M1\}$ to itself ?

4. You have to choose at random an ordered choice of 4 lectures from $\{M1F, M1S, M1GLA, M1M1\}$ with replacement. What is the probability that the choice will have no M1F lectures?

5. Evaluate the limit:
$$\lim_{x \to 0} \left[\frac{\cos(x) - \exp(x) + x}{\sin(x^2)} \right]$$

6. If $u = (1, 2, a)$ and $v = (-3, a, 1)$ then $u \times v$ is

   (a) $(0, 0, 0)$ (b) $(2 - a^2, 3a + 1, a + 6)$ (c) $(2 - a^2, -3a - 1, a + 6)$

7. Two squares have side lengths, X and Y, in metres, which are continuous random variables with joint pdf:
$$f_{X,Y}(x, y) = \begin{cases} 10xy^2 & 0 < x \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$
   What is the expected total area of the two squares?

8. How many symmetric, reflexive relations are there on {Liebeck, McCoy, Mestel, Thomas} with Mestel $\sim$ Thomas ?

Questions D

1. Suppose $y = x^3$, $u = dy/dx$ and $v = du/dx$. What is xdv/du?

2. Suppose that $x > y$.

   (a) $\Rightarrow xz > yz$.

   (b) $\Rightarrow -xz < -yz$.

   (c) Now put $z = -x$ to find that $x^2 < xy$.

   (d) But if we put $x = 2, y = 1$ this gives a contradiction.

   At which step (a,b,c or d) did the argument go wrong ?

3. If $y(x) = x^2 e^{-2x}$, what is the value of $y^{(100)}(0)$?

4. Let H be the set of hairs on Prof Mestel's beard. Is it possible to define addition and scalar multiplication in such a way as to make H a vector space over $\mathbb{R}$?

   (a) Yes (b) No

5. Professor Mestel has 5^{5^5} hairs in his beard. He pulls out one per hour, the first at 1am. Assuming he lives a very long time, at what time of day will his face first look like a baby's bottom ?

6. Professor Mestel has 5^{5^5} hairs in his beard. The time is takes to pull out a hair (in hours) is exponentially distributed with paramter 0.2. Given that the hair pulling out process starts at 8pm and assuming that he lives a very long time, what time of day do you expect his face to first look like a baby's bottom ?

7. True or false: we can define a vector product on $\mathbb{R}^4$ as follows – for $a = (a_1, a_2, a_3, a_4)$ and $b = (b_1, b_2, b_3, b_4)$, define $a \times b = \det \begin{pmatrix} e_1 & e_2 & e_3 & e_4 \\ a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \end{pmatrix}$.

   (a) True (b) False

8. The number of mince pies I will eat today has pgf given by:

$$G(t) = \frac{1}{2^8}(1+t)^8$$

   How many mince pies do I expect to eat?

JMC Questions A

1. Thomas is happy only if Mestel is unhappy. Suppose Mestel is happy. Is Thomas

   (a) Definitely happy, (b) Possibly happy or unhappy, or (c) Definitely unhappy ?

2. How many of the following statements are true?

   (i) $\sin(2x^2)$ is periodic with period $\sqrt{\pi}$

   (ii) $\cos|x|$ is 2π-periodic

   (iii) $|\cos x|$ is π-periodic

   (a) None true (b) one true (c) two true (d) all true

3. The maths department at the University of Impioxcam consists of k very unfriendly lecturers, where $k \geq 4$. Each lecturer is friends with exactly 2 other lecturers. For any 2 lecturers who aren't friends, there is exactly 1 other lecturer who is friends with both of them. The total number k is lecturers is

   (a) 4 (b) 5 (c) 6 (d) could be any integer ≥ 4

4. How many (complex) solutions are there to the equation

$$|z| = \Re e[z] + \Im m[z],$$

   where $\Re e$ and $\Im m$ denote real and imaginary parts?

   (a) As many times as Martin Liebeck has climbed Mount Everest.

   (b) As many times as Richard Thomas gave a M1F lecture on a Thursday this term.

   (c) More times than Emma McCoy has had hot dinners.

   (d) None of these.

5. What's

$$\sup \bigcap_{n=1}^{\infty} \left(1 - \frac{1}{n}, 1\right) ?$$

6. What are the roots of $1 - z + z^2 - z^3 + - z^{2n-1}$? $e^{\frac{k\pi i}{n}}$ $k = 1 .. 2n-1$.

7. True or false: the sum of the eigenvalues of the matrix $\begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}$ does not depend on a or b.

   (a) True (b) False

8. The function $f(x) = x \sin(1/x)$ for $x \neq 0$, $f(0) = 0$ is

   (a) an odd function and is continuous for all x.

   (b) a periodic function with period 0.

   (c) discontinuous at $x = 0$ and $f(x) \to 1$ as $x \to \infty$

   (d) an even function with at least three roots.

$Q^TQ = I$

$x = \lambda \cdot 12 + 20 \cdot \mu$

$\begin{pmatrix} a & -b \\ 0 & b \end{pmatrix}\begin{pmatrix} a & 0 \\ -b & b \end{pmatrix} = \begin{pmatrix} a^2+b^2 & -b^2 \\ -b^2 & b^2 \end{pmatrix}$

JMC Questions B

1. True or false: the matrix $\begin{pmatrix} a & -b \\ 0 & b \end{pmatrix}$ can never be orthogonal.

$\begin{matrix} a^2 b^2 & b^2 \\ -b^2 & b^2 \end{matrix} = \begin{pmatrix} a & -b \\ 0 & b \end{pmatrix}\begin{pmatrix} a & 0 \\ -b & b \end{pmatrix}$

(a) True (b) False

2. In the specially designed Maths department snack machine you can use positive and negative numbers of 12p and 20p coins. It accepts no other coins, and charges a positive amount for all items. What's the least it can charge for any snack ?

4

3. The integral

$$I = \int_{-\infty}^{\infty} \frac{\sin(x + x^3)}{1 + 3x^2}\, dx, \qquad \text{is}$$

$20 = 1 \cdot 12 + 8$

$12 = 1 \cdot 8 + 4$

$8 = 2 \cdot 4$

(a) $2\cos(\infty)$ (b) 0 (c) does not exist (d) $1 < I < \tan^{-1}(1/\sqrt{3})$.

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5. McCoy and Thomas are both giving the same course, with lectures timetabled at the same times, on "Fundaments of Analistics" to a class of 7 students. If a student is in Thomas's lecture one day, she (or he) is so baffled that the next day she goes to McCoy's lecture. If a student is in McCoy's lecture one day, she is inspired but out of sympathy she throws a die, and if she gets a 6 goes to Thomas next day, otherwise to McCoy again. At the 277^{th} lecture of the course, Thomas can expect an attendance of

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(a) Injective (b) Surjective (c) Bijective (d) More than one of these (e) None of these

Is your condition sufficient ?

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(a) an ellipse (b) a parabola (c) a hyperbola (d) degenerate

JMC Questions C

1. Let C be the set of points in $\mathbb{R}^2$ on the unit circle $x^2 + y^2 = 1$. Is it possible to define addition and scalar multiplication in such a way as to make C a vector space over $\mathbb{R}$?

   (a) Yes (b) No

2. The infinite series

$$S = \sum_{n=1}^{\infty} \frac{x^n}{n}$$

   (a) $S = -\log(1-x)$ provided $|x|$ is small enough ✓
   (b) has infinite radius of convergence ✗
   (c) describes a function whose n'th derivative $f^{(n)}(0) = 1/n$ ✗
   (d) none of these

3. How any functions are there from $S := \{M1F, M1S, M1GLA, M1M1\}$ to itself ? 4^4

4. True or false: the set of all sets which do not belong to themselves does not belong to itself.

   (a) True (b) False (c) Neither true nor false

5. Evaluate the limit:

$$\lim_{x\to 0}\left[\frac{\cos(x) - \exp(x) + x}{\sin(x^2)}\right] = -1$$

6. If $u = (1, 2, a)$ and $v = (-3, a, 1)$ then $u \times v$ is

   (a) $(0,0,0)$ (b) $(2-a^2, 3a+1, a+6)$ (c) $(2-a^2, -3a-1, a+6)$

7. How many equivalence relations are there on the set of natural numbers ?

   ~~(a) None~~
   ~~(b) Finitely~~ many
   (c) Countably infinite many
   (d) Uncountably infinitely many ✓

8. How many symmetric, reflexive relations are there on {Liebeck, McCoy, Mestel, Thomas} with Mestel $\sim$ Thomas ? 2^9

$-\sin(x) - e^x + 1$ / $\cos(x^2)$

$4 - 2$ / 2

$a \sim b \Leftrightarrow b \sim a$ $= 2^{\frac{4\times 3}{2}}$ $= 2^6$

$a \sim a$ $= 2^{(n\times n - n)} = 2^{2\times 2 - 2}$ 2^5

1 2 a
-3 a 1

2^9

JMC Questions D

1. Suppose $y = x^3$, $u = dy/dx$ and $v = du/dx$. What is xdv/du?

2. Suppose that $x > y$.

   (a) $\Rightarrow xz > yz$.

   (b) $\Rightarrow -xz < -yz$.

   (c) Now put $z = -x$ to find that $x^2 < xy$.

   (d) But if we put $x = 2, y = 1$ this gives a contradiction.

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   (a) Yes (b) No

5. Professor Mestel has 5^{5^5} hairs in his beard. He pulls out one per hour, the first at 1am. Assuming he lives a very long time, at what time of day will his face first look like a baby's bottom ?

6. Liebeck always proves a new theorem on the nth day of the week. Thomas always proves a theorem on the n^7th day of the week, and notices it's the same day of the week as Liebeck. What are the possible values of n ?

7. True or false: we can define a vector product on $\mathbb{R}^4$ as follows – for $a = (a_1, a_2, a_3, a_4)$ and $b = (b_1, b_2, b_3, b_4)$, define $a \times b = \det \begin{pmatrix} e_1 & e_2 & e_3 & e_4 \\ a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \end{pmatrix}$.

   (a) True (b) False

8. What is the set $\bigcap_{k=1}^{\infty}\{m/kn \in \mathbb{Q} : m, n \in \mathbb{Z}\}$?

$5^{5^5} =$

$5 = 5 \mod 24$

$5^5 = 5^5 \mod 24$

$= 5 \mod 24$

$5^5 = 5^5 \mod 24$

$5^2 = 1 \mod 24$

$5^1 = 5 \mod 24$

$5^{a+b} = 5^a \cdot 5^b$

$5^5 = 3125$

Solutions A

1	(c)
2	$\frac{1}{31}$
3	(b)
4	(c) (infinitely many)
5	$\frac{2048}{2047}$
6	$\exp(2k\pi i/2n)$ where k is any integer from 0 to $2n-1$ except n.
7	True
8	(d) (even function with at least 3 roots)

Solutions B

1	$\frac{2}{3}$
2	4p
3	$\frac{36}{5} = 7\frac{1}{5}$
4	(d) (None; normal limit)
5	(b)
6	True (diffble means integrable)
7	(a) and yes it is sufficient
8	(c)

Solutions C

1	(a)
2	(a) $-\log(1-x)$
3	$4^4 (= 256)$
4	$\left(\frac{3}{4}\right)^4$
5	-1 (limit)
6	(c)
7	$\frac{15}{14}$
8	~~3~~ 32

Solutions D

1	1 (xdv/du)
2	(a)
3	9900×2^{98} (I screwed up; 2^{98} wasn't meant to be there!)
4	(b)
5	5am
6	9pm
7	(b)
8	4

JMC Solutions A

1	(c)
2	(c) (two true, $\cos\lvert x\rvert$ etc)
3	(b)
4	(c) (infinitely many)
5	Doesnt exist (or $-\infty$).
6	$\exp(2k\pi i/2n)$ where k is any integer from 0 to $2n-1$ except n.
7	True
8	(d) (even function with at least 3 roots)

JMC Solutions B

1	(a)
2	4p
3	(b) 0 (odd integral)
4	(d) (None; normal limit)
5	(b)
6	True (diffble means integrable)
7	(a) and yes it is sufficient
8	(c)

JMC Solutions C

1	(a)
2	(a) $-\log(1-x)$
3	$4^4 (= 256)$
4	(c)
5	-1 (limit)
6	(c)
7	(d)
8	~~2~~ 32.

JMC Solutions D

1	1 (xdv/du)
2	(a)
3	9900×2^{98} (I screwed up; 2^{98} wasn't meant to be there!)
4	(b)
5	5am
6	~~False~~ Ung N'
7	~~(b)~~ False
8	$\mathbb{Q}$