Name (IN CAPITAL LETTERS!): ....

CID: ....

Question 2. Mr S. and Mr T. have a pancake-making competition. Naturally, being eminent pure mathematicians, they don't actually go into the kitchen until they have defined precisely what a pancake is, and have used advanced geometrical techniques to prove that it is indeed possible to construct one.

The numbers of pancakes each makes, $S(t)$ and $T(t)$, may be treated as real numbers and are once-differentiable functions of time in $0 < t < 2$, and are of course continuous in $0 \leqslant t \leqslant 2$.

When they start ($t = 0$) neither has made any pancakes, while after 2 hours ($t = 2$), Mr S. has produced twice as many pancakes, as has Mr. T.

(a) By considering a suitable combination of S and T, prove that there must exist a time t_0 at which $dS/dt = 2dT/dt$.

(b) If Mr S.'s winning formula involves

$$\frac{dS}{dt} = \frac{60}{3 + \cos(t^2)},$$

use the Mean Value Theorem to prove that at $t = 2$, Mr S. has produced between 30 and 60 pancakes. *[Do not try to find $S(t)$ exactly.]*

Answer. Defining the function $f(t) = S(t) - 2T(t)$, we see $f(0) = 0 = f(2)$. Furthermore, $f(t)$ is continuous and differentiable. Therefore by Rolle's Theorem (or the MVT) there exists a time t_0 in $0 < t_0 < 2$ such that $f'(t_0) = 0$. It follows that at $t = t_0$, $S'(t_0) = 2T'(t_0)$.**(5 marks)**

The MVT states that for some ξ in $0 < \xi < 2$,

$$\frac{S(2) - S(0)}{2 - 0} = f'(\xi) = \frac{60}{3 + \cos \xi^2}$$

Now as $|\cos \xi^2| \leqslant 1$,

$$\tfrac{60}{4} \leqslant \frac{60}{3 + \cos \xi^2} \leqslant \tfrac{60}{2}$$

As $S(0) = 0$, we have

$$15 \leqslant \tfrac{1}{2}S(2) \leqslant 30$$

so that $30 \leqslant S(2) \leqslant 60$ as required.**(5 marks)**