Name (IN CAPITAL LETTERS!): ....

CID: ....

Question 3. The function f is given by $f(x) = \log(1 + \sin x)$.

- (a) For which values of x is this NOT a valid definition?
- (b) If $f(x) = g(x) + h(x)$ where $g(x)$ is even and $h(x)$ is odd, then express $g(x)$ in the simplest form you can.
- (c) For which values of x is $g(x)$ defined?
- (d) Prove that $f(1) > g(1)$.

You are reminded that the function sin *takes radians – real mathematicians do not use degrees.*

Answer.

- (a) $f(x)$ is defined unless the argument of the logarithm is zero. This happens if $\sin x = -1$, or $x = (2n - 1/2)\pi$, where n is any integer. **(2 marks)**
- (b) $g(x) = \frac{1}{2}(\log(1+\sin(x))+\log(1+\sin(-x))) = \frac{1}{2}\log(1-\sin^2 x) = \log(|\cos x|)$ **(3 marks)**. Deduct 1 for missing out the modulus, deduct 1 or 2 for insufficient simplification.
- (c) $g(x)$ is defined unless $\cos x = 0$ i.e. for $x \neq (n + \frac{1}{2})\pi$ **(2 marks)**
- (d) As $0 < 1 < \pi$, $\sin(1) > 0$, $1 + \sin 1 > 1$ and so $\log(1 + \sin(1)) > 0$. As $|\cos(1)| < 1$, $\log|\cos(1)| < 0$. Therefore $f(1) > 0 > g(1)$. **(3 marks)**