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Question 4. Three stand-up comedians "Tommy O'Clicker", "Becks" and "Mac the Knife" are performing at the Empire Theatre between $x = 0$ and $x = 1$. The popularity of each is measured by the Sound Of Laughter Evaluation (SOLE) survey, and is given at time x by the three functions

$$T(x) = \sqrt{2}\sin(3\pi x/4), \qquad B(x) = x(2-x)^2 \qquad \text{and} \qquad M(x) = x(4-3x).$$

Determine the maximum popularity of each comedian over the interval $(0,1)$. Who is most popular at their peak?

Sketch these functions carefully on the same diagram over the interval $(0,1)$, indicating clearly which is which.

You may find it helpful to note that $\frac{3}{4}\pi\sqrt{2} \simeq 3.33$.

Answer. $T(x)$ clearly takes the maximum value of $\sqrt{2}$ when $x = 2/3$. **(1 mark)**

$B' = 4 - 8x + 3x^2 = (2-3x)(2-x) = 0$ when $x = 2/3$ in the range $(0,1)$. $B'' = -8 + 6x < 0$ when $x = 2/3$. So this is a maximum. When $x = 2/3$, $B = 32/27$. **(2 marks)**

$M' = 4 - 6x = 0$ when $x = 2/3$ (again). This is a maximum, and $M = 4/3$. **(1 mark)**

All maxima are at the same x-value. Clearly $4/3 = 36/27 > 32/27$. Also $(4/3)^2 = 16/9 < 2$, so we conclude $\sqrt{2} > 4/3 > 32/27$, i.e. $T > M > B$ at $x = 2/3$. **(2 marks)**

Sketch: All curves go through $(0,0)$ and $(1,1)$ **(1 mark)**

$B'(0) = M'(0) = 4$. $T'(0) = \frac{3}{4}\pi\sqrt{2} \simeq 3.33 < 4$ (given). Thus for small x $T < B, M$, but eventually it becomes larger. so curves must cross somewhere. (This was hinted at in question so sketch should illustrate it. Penalise unless some crossings shown clearly) **(1 mark)**

General shape of curves **(2 marks)**