

M1M1 Progress Test 3: December 1st 2008.

Write your name **clearly** on your answer book.

No calculators, books or lecture notes. 50 minutes. Attempt all 4 questions.

1. The function $y(x)$ obeys the equation

$$y'' + xy = 0 \quad \text{with} \quad y(0) = 1, \quad y'(0) = 0. \quad (1)$$

What are the values of $y''(0)$ and $y'''(0)$?

Use de l'Hôpital's rule to evaluate the limit

$$\lim_{x \rightarrow 0} \left[\frac{y(x) - 1}{\sin^3 x} \right].$$

2. Differentiate equation (1) n times, and hence find the Maclaurin series for $y(x)$, giving the first 3 non-zero terms explicitly.

Use the ratio test to determine for which values of x the series converges.

3. Use the Mean Value Theorem and standard properties of e^x to prove that

$$1 > \frac{1 - e^{-x}}{x} > e^{-x} \quad \text{for} \quad x > 0.$$

Deduce that

$$\frac{1}{1+x} > e^{-x} > 1-x \quad \text{for} \quad x > 0. \quad (2)$$

4. Sketch carefully and clearly in the same diagram the three functions in the inequalities of (2).

Verify that (2) holds in $x > 0$. How does it require modification for $x < 0$?