

M1M1 Progress Test 3: December 7th 2007.

Write your name **clearly** on your answer book.

No calculators, books or lecture notes.

50 minutes. Attempt all four questions.

1. Three curves are defined as follows:

(a)
$$y^2 = \frac{(x^2 - 1)^2}{2 - x^2}.$$

(b) In terms of usual polar coordinates

$$r^2(1 \pm \sin \theta) = 1.$$

(c) The set of all complex numbers z satisfying

$$z\bar{z}(8 - (z + \bar{z})^2) = 4.$$

where \bar{z} is the complex conjugate of z .

Prove that these three curves are identical.

2. Show that yet another representation of the curve in question 1 is

$$y = \pm \left(\sqrt{(2 - x^2)} - \frac{1}{\sqrt{2 - x^2}} \right).$$

Sketch the curve using any convenient form and identify any features of interest.

3. Given that n is an integer and $n > 1$, evaluate the integral

$$\int_0^{2\pi} (x + x^2)e^{inx} dx.$$

Hence show that

$$\int_0^{2\pi} (x + x^2) \cos 6x dx = \frac{1}{9}\pi.$$

4. Evaluate the integrals

$$\int_0^x \frac{e^t}{\cosh t} dt \quad \text{and} \quad \int_0^x \frac{dy}{1 + \sin y}.$$

For which values of $x \geq 0$ do the integrals exist?