1. Three curves are defined as follows:

(a)
$$y^2 = \frac{(x^2 - 1)^2}{2 - x^2}.$$

(b) In terms of usual polar coordinates
$$r^2(1 \pm \sin\theta) = 1.$$

(c) The set of all complex numbers z satisfying
$$z\bar{z}(8 - (z + \bar{z})^2) = 4.$$

where $\bar{z}$ is the complex conjugate of z.

Prove that these three curves are identical.

2. Show that yet another representation of the curve in question 1 is
$$y = \pm\left(\sqrt{(2 - x^2)} - \frac{1}{\sqrt{2 - x^2}}\right).$$

Sketch the curve using any convenient form and identify any features of interest.

3. Given that n is an integer and $n > 1$, evaluate the integral
$$\int_0^{2\pi} (x + x^2)e^{inx}\, dx.$$

Hence show that
$$\int_0^{2\pi} (x + x^2)\cos 6x\, dx = \tfrac{1}{9}\pi.$$

4. Evaluate the integrals
$$\int_0^x \frac{e^t}{\cosh t}\, dt \qquad \text{and} \qquad \int_0^x \frac{dy}{1 + \sin y}.$$
For which values of $x \geqslant 0$ do the integrals exist?