M1M1 Progress Test 3: December 8^{th} 2006.

Write your name **clearly** on your answer book.

No calculators, books or lecture notes.

50 minutes. Attempt all four questions.

1. (a) Sketch the curve $y = x^2 e^{-x}$. You should determine any stationary points precisely but you need not find any inflection points.

(b) The function $f(x) = (x^2 + 8x + 20)e^{-x}$ is approximated by its Maclaurin series up to and including terms proportional to x^3:

$$f(x) = 20 - 12x + 3x^2 - \tfrac{1}{3}x^3 + E_4.$$

Use Taylor's theorem to estimate the maximum possible value of the error E_4 if $3 > x > 0$.

2. Find all complex solutions of the equation $\sin z = 4$.

3. Plot the following curves in the complex z-plane, if the argument of complex numbers is defined to lie between 0 and 2π.

$$(a) \quad \mathrm{Im}(z) = \tfrac{1}{2}\pi \qquad (b) \quad |z| = \tfrac{1}{2}\pi$$

$$(c) \quad \arg(z) = \tfrac{1}{2}\pi \qquad (d) \quad |z|\arg(z) = \tfrac{1}{2}\pi$$

4. Calculate the n'th derivative of the function

$$f(z) = \frac{1}{z-1}$$

evaluated at the point $z = 3$. Hence write down the Taylor series of $f(z)$ about $z = 3$ and give its radius of convergence.

Sketch the region in the complex z-plane for which this Taylor series converges to $f(z)$. [You need not worry about the behaviour on the edge of this region.]