
M1M1: Progress Test 3: December 9th 2005

Write your name **clearly** on your answer book.

No calculators. No books/lecture notes.

50 minutes. Attempt all four questions.

1. The function $f(x)$ is defined to be

$$f(x) = xe^{-x}.$$

- (a) Find the first three non-zero terms in the Taylor series expansion of $f(x)$ about $x = 0$;
- (b) Use Taylor's theorem to find an estimate of the maximum error incurred in using these first three terms in the Taylor series to approximate $f(x)$ in the interval $0 \leq x \leq 1$.

2. Find all complex solutions of the following equations:

(a) $z^3 + (1 - i)z^2 + (1 - i)z - i = 0$;

(b) $\coth z = 2$.

3. Let two complex variables ζ and z be related by the formula

$$z = \frac{\zeta + i}{\zeta - 2}.$$

Find the curve, in the complex z -plane, corresponding to the image of the circle $|\zeta - 1| = 1$. Sketch this curve.

4. Compute the following indefinite integrals:

$$(a) \int \frac{dx}{x^2 + 2}; \quad (b) \int \frac{dx}{x^2 - 2}; \quad (c) \int \frac{dx}{x^3 - x^2 + x - 1}.$$

THE END