M1M1: Progress Test 3: December 6th 2004

Write your name **clearly** on your answer book.
No calculators. No books/lecture notes.
50 minutes. Attempt all **five** questions.

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1. Sketch a graph of the function

$$y(x) = \frac{2x^2 + 1}{x - 1}$$

carefully indicating on your sketch any important features.

2. Show that

$$\frac{1}{2} < \log 2 < 1.$$

3. Use Taylor's theorem to find an estimate for the maximum possible error incurred in approximating the function $\log(1 + x)$ by the polynomial

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$$

for values of x such that $|x| \leq \frac{1}{2}$.

4. Find all complex solutions z to the following equations:

$$(a)\ \tanh z = 2;$$
$$(b)\ \tanh |z| = 2;$$
$$(c)\ z + \frac{1}{\bar{z}} = 2.$$

5. Evaluate the following integrals:

$$(a)\ \int \frac{x+1}{x-1}\, dx;$$
$$(b)\ \int \frac{x^2+1}{x^2-1}\, dx;$$
$$(c)\ \int \frac{x^2-1}{x^2+1}\, dx.$$

THE END