M1M1: Progress Test 3: December 12th 2003

Write your name and tutorial group number on your answer book.
No calculators. No books/lecture notes.
50 minutes. Attempt all four questions.

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1. Use Taylor's theorem to find an estimate of the maximum error involved in using the polynomial $1-x+x^2-x^3$ to approximate the value of $1/(1+x)$ for values of x in the interval $0 \leq x \leq 1/2$.

2. Find all complex solutions to the following equations:

$$\text{(a) } z^3 = -i; \qquad \text{(b) } \tanh z = 3.$$

Sketch the locus, in the complex plane, of all the z-values satisfying

$$\arg\left[\frac{z-i}{z+i}\right] = 0 \text{ or } \pi, \quad z \neq \pm i.$$

3. Find explicit formulae for the following indefinite integrals:

$$\int \frac{e^x}{e^x+2}\,dx; \qquad \int xe^{2x}\,dx; \qquad \int \frac{dx}{x^2+x+1}.$$

4. Let n denote any non-negative integer and define

$$I_n = \int_0^1 x^n \log x \; dx,$$
$$J_n = \int_0^1 x(\log x)^n \; dx.$$

Find the values of I_{100} and J_{100}.

THE END