
M1M1: Progress Test 3: December 9th 2002

Write your name and tutorial group number on your answer book.

No calculators. No books/lecture notes.

50 minutes. Attempt all three questions.

1. Find all complex solutions of the following equations and, in each case, indicate where the solutions are in the complex plane by means of a sketch:

(i)

$$z^3 - (3 - i)z^2 + (2 - 3i)z + 2i = 0;$$

(ii)

$$z^2 \bar{z}^2 + z \bar{z} - 6 = 0;$$

(iii)

$$\cos |z| = -1;$$

(iv)

$$\tanh z = 2.$$

2. It is known that $\cos 4\theta$ can be written as a polynomial in $\cos \theta$, i.e.

$$\cos 4\theta = c_4 \cos^4 \theta + c_3 \cos^3 \theta + c_2 \cos^2 \theta + c_1 \cos \theta + c_0.$$

Use De Moivre's theorem to determine the 5 coefficients $c_j, j = 0, 1, \dots, 5$.

3. Calculate the values of the definite integrals:

$$\int_0^1 \frac{(x-1)^2}{x^2 + 4x + 3} dx.$$

$$\int_0^1 \sqrt{\frac{x}{2-x}} dx;$$

$$\int_0^\pi \sin x \cos^4 x dx.$$

THE END