

M1M1 Progress Test 2: November 10th 2008.

Write your name **clearly** on your answer book.

No calculators, books or lecture notes. 50 minutes. Attempt all four questions.

1. Expand $\exp(x^2)$ as a series in $(x - 1)$, neglecting terms of $O(x - 1)^3$.

2. Find, if possible, the following limits

$$(a) \quad \lim_{x \rightarrow 1} \left[\frac{\log x}{\sin \pi x} \right] \quad (b) \quad \lim_{x \rightarrow 0} x^x$$
$$(c) \quad \lim_{x \rightarrow \infty} \left[e^{-x^2} \tan x \right] \quad (d) \quad \lim_{x \rightarrow \infty} \left[x^2 \left((1 + x^3)^{1/3} - x \right) \right].$$

3. Find, *from first principles*, the derivative of $1/x^2$.

(**N.B.** Do **not** assume the binomial series for $(1 + x)^n$ unless n is a positive integer.)

4. If $y = A/(x + b)$ for some constants A and b , obtain a formula for the n 'th derivative

$$F \equiv \frac{d^n y}{dx^n} \quad \text{in terms of } x, A, b \text{ and } n.$$

Regarding x as a function of y , find a similar formula for

$$G \equiv \frac{d^n x}{dy^n}.$$

When, if ever, does $F = 1/G$?