M1M1 Progress Test 2: November 16^{th} 2007.

Write your name **clearly** on your answer book.

No calculators, books or lecture notes.

50 minutes. Attempt all four questions.

1. The (continuous) function $f(x)$ is defined for all x by

$$f(x) = x^2 \sin\left(\frac{1}{x}\right) \qquad \text{for } x \neq 0, \quad \text{and} \quad f(0) = 0.$$

(a) Using any method, find $f'(x)$, for $x \neq 0$.

(b) For $x = 0$, show from first principles that the derivative $f'(0)$ exists and evaluate it.

(c) Combining (a) and (b), what do you deduce about the derivative $f'(x)$?

2. Using Leibniz' rule, evaluate the n'th derivative

$$\left(\frac{d^n}{dx^n}\right)[x^2 e^x].$$

Hence find the Maclaurin series for the function $x^2 e^x$.

Compare your answer with one obtained directly from the series for $\exp(x)$.

3. The function $g(x)$ is such that $g(0) = 0$ and has the derivative

$$g'(x) = \frac{1}{5 + \sin x}.$$

Use the Mean Value Theorem to find numbers α and β such that

$$\alpha < g(\tfrac{1}{2}\pi) < \beta.$$

4. Determine, by any method, the value of the limit

$$\lim_{t\to\infty}\left[\frac{\log(1+t)}{t} + \frac{t^2+7t+12}{t^3+4t^2-1} + (\tanh t)\tan^{-1}(2t) + t\sin\left(\frac{2}{t}\right)\right].$$