M1M1 Progress Test 2: November 17^{th} 2006.

Write your name **clearly** on your answer book.

No calculators, books or lecture notes.

50 minutes. Attempt all four questions.

1. The function $f(x)$ is given by

$$f(x) = \cos(x^{-1}) + (\cos x)^{-1} + \cos^{-1}(x) \ .$$

For what values of x is $f(x)$ defined?

Using any method, find the derivative $f'(x)$.

2. Find, **from first principles** (i.e. from the definition as a limit), the derivative of $\sqrt{1+\exp(x)}$.

[You may use the series definition and standard properties of $\exp(t)$. You might find the identity $a - b = (a^2 - b^2)/(a + b)$ helpful.]

3. Use Leibniz' formula to find

$$\frac{d^{20}}{dx^{20}} \left[(1 + x + x^2) \sin x\right] \ .$$

4. The point (x, y), with $x > 0$ and $y > 0$, lies on the ellipse $2x^2+y^2 = 1$. A rectangle is formed by joining the four points $(\pm x, \pm y)$.

Find the area of this rectangle as a function of x.

Show that the maximum possible area is $\sqrt{2}$.