M1M1: Progress Test 2: November 15th 2004

Write your name **clearly** on your answer book.

No calculators. No books/lecture notes.

50 minutes. Attempt all **five** questions.

1. Compute the following three limits:

$$(a)\ \lim_{x\to 1}\left(\frac{x^4-1}{x-1}\right);$$

$$(b)\ \lim_{x\to 0}\left(\frac{\sqrt{1+2x}-1}{x}\right);$$

$$(c)\ \lim_{x\to\infty}\left(\frac{2+3e^x}{1+4e^x}\right).$$

2. Find the first three non-zero terms in the series expansion, in powers of x, of the three functions:

$$(a)\ e^{\sin x};$$

$$(b)\ \sin(x+x^2);$$

$$(c)\ \frac{1}{1+\sqrt{1+x}}.$$

3. Find the derivative of $e^{\sqrt{x}}$ from first principles.

4. Using any method, find the derivatives of the following three functions:

$$(a)\ 4^x;$$

$$(b)\ \log(1+e^{2x});$$

$$(c)\ \tan(x^x).$$

5. If the function $f(x)$ is defined as

$$f(x) = x^2 e^{-2x},$$

find the value of $f^{(n)}(0)$ (that is, the value of the n-th derivative of $f(x)$ evaluated at $x = 0$).

THE END