M1M1: Progress Test 2: November 14th 2003

Write your name and tutorial group number on your answer book.
No calculators. No books/lecture notes.
50 minutes. Attempt all five questions.

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1. Compute the following limits:

$$\lim_{x\to 0}\left[\frac{\tanh x}{x}\right]; \quad \lim_{x\to \pi/2}\left[\frac{\sin x - 1}{\cos x}\right]; \quad \lim_{x\to 0}\left[\frac{\sqrt{(1+\sqrt{x})}-\sqrt{(1-\sqrt{x})}}{\sqrt{x}}\right].$$

2. By writing

$$\log(1+x) = a_1 x + a_2 x^2 + a_3 x^3 + ...$$

and using the fact that

$$1 + x = \exp(\log(1+x)), \quad \text{for } |x| < 1,$$

find a_1, a_2 and a_3.

Using this result, or otherwise, compute the three limits:

$$\text{(a)} \lim_{x\to 0}\left[\frac{\log\cos x}{x^2}\right]; \quad \text{(b)} \lim_{x\to 0}\left[\frac{\log(1+x^2)}{\sin(2x)}\right]; \quad \text{(c)} \lim_{x\to 0}\left[(\cos x)^{1/x}\right].$$

3. Find the derivative of $\sin\sqrt{x}$ from first principles.

4. Find the derivative of the following functions:

$$\text{(a) } (x^{10}+1)^{1/2}; \quad \text{(b) } x\tanh(x); \quad \text{(c) } \frac{x}{e^x + e^{-x}}; \quad \text{(d) } [\exp(x)]^x.$$

5. Find an explicit expression for

$$\frac{d^n}{dx^n}\left(x^2 e^{3x}\right)$$

where $n \geq 2$ is a positive integer.

THE END