Solutions and mark scheme for M1M1 Progress Test 1:

1. We have

$$h(x) = \left[\frac{1}{1+x} - 1\right]^2 \left(\frac{1}{1+x}\right)^{-2} = \frac{(1-1-x)^2}{1} = x^2$$

cancelling a factor of $(x+1)^2$, which isn't valid if $x = -1$. The definition of $f(x)$ requires $x \neq -1$, that of $g(x)$ requires $x \neq 0$. The definition of $h(x)$ requires $x \neq -1$ and $g(x) \neq 0$. This last never happens, so $h(x)$ is defined for $x \neq 1$.

2. Sketch below. $f(x)$ takes all values between $\cos 1$ and $\cos 0 = 1$, so the range of $f(x)$ is $\cos 1 \leqslant x \leqslant 1$.

$f(x)$ is even, because $f(-x) = f(x)$.

$f(x)$ is π-periodic, because $f(x+\pi) = \cos[\cos(x+\pi)] = \cos(-\cos x) = \cos(\cos(x))$

3. The domain of $\sin^{-1} x$ is $-1 \leqslant x \leqslant 1$.

If $y = 1 + e^x$, then $x = \log(y-1)$. So $g^{-1}(x) = \log(x-1)$.

Thus, $g^{-1}(x)$ takes real values for $x > 1$, so this is the domain of g^{-1} (it is of course the *range* of g also.)

Now $\sin^{-1}[g^{-1}(x)]$ requires $x > 1$ and also $-1 \leqslant g^{-1}(x) \leqslant 1$. So we must have $-1 \leqslant \log(x-1) \leqslant +1$ or $1 + 1/e < x < 1 + e$.

4.

(a) Wrong. $fg = \cos^2 x - \sin^2 x$ which is even, not odd.

(b) Wrong. The function $h(x) = 3$ is an even function of x, as it obviously stays the same when $x \to -x$. (though it is always an *odd* integer – don't confuse the two ideas).

(c) Correct. $fg \equiv 0$ is both an odd and an even function.

(d) Correct. $f + g = x^2 + 1$ is even. $fg = x(x^2 - 1)$ is odd, as required.

(e) Wrong. $fg = g(x)g(-x)$ which is even, not odd. [Note (a) is a special case of (e)]

Total : 40

General instructions to markers

Each script should be awarded an integer mark between 0 & 20 inclusive. Take the total out of 40 and divide by 2, rounding up or down using your judgement.

You should deduct marks for illegibly named scripts and may penalise mathematical incoherence. Well presented arguments may deserve credit even if marred by algebraic or arithmetical slips. You may award a bonus mark for 'good' mathematics. Correct, but unexplained answers may not deserve full credit, at your discretion. Not everything in the model answers is needed for full credit.