M1M1 Progress Test 1: October 20^{th} 2008.

Write your name **clearly** on your answer book.

No calculators, books or lecture notes. 50 minutes. Attempt all four questions.

1. Find and simplify the composite function $h(x) = f[g(x)]$ where

$$f(x) = \frac{(x-1)^2}{x^2} \qquad \text{and} \quad g(x) = \frac{1}{1+x}.$$

For which values of x can each of the functions f, g and h be defined in this way?

2. Give a rough sketch of the graph $y = f(x)$ where the function

$$f(x) = \cos(\cos x).$$

What is the range of f as x takes all real values?

Is $f(x)$ odd, even, or neither?

Is $f(x)$ periodic? If so, state its period.

3. What is the largest pssible domain of the inverse trigonometrical function $\sin^{-1}(x)$?

Find the inverse function, $g^{-1}(x)$ for the function $g(x) = 1 + e^x$.

What is the domain of g^{-1}?

What is the domain of the function $h(x) = \sin^{-1}[g^{-1}(x)]$?

4. Last year I asked students to find me two functions, $f(x)$ and $g(x)$, such that the product $f(x)g(x)$ was an odd function of x, and the sum $f(x)+g(x)$ was an even function. I received the following answers:

(a) $f = \cos x + \sin x$, $g = \cos x - \sin x$
(b) $f = 1$, $g = 3$
(c) $f = 0$, g is any even function
(d) $f = x + 1$, $g = x^2 - x$.
(e) $f(x) = g(-x)$, g is any function.

Which, if any, of these answers is correct? Give a brief explanation of any wrong answers.