M1M1: Progress Test 1: October 28th 2005

Write your name **clearly** on your answer book.
No calculators. No books/lecture notes.
50 minutes. Attempt all four questions.

---

1. The function $f(x)$ is defined as

$$f(x) = \exp(x + x^2)$$

(a) $f(x)$ can be written as the sum of an even function $f_e(x)$ and an odd function $f_o(x)$ so that

$$f(x) = f_e(x) + f_o(x).$$

Find $f_e(x)$ and $f_o(x)$ explicitly.

(b) Show that

$$(f_e(x))^2 - (f_o(x))^2 = \exp(2x^2).$$

2. Put the following two rational functions in partial fraction form:

$$(a)\ \frac{x^2}{x^2-1};\quad (b)\ \frac{1}{x^3-1}.$$

3. A function $f(x)$ is defined to be

$$f(x) = \frac{1}{\exp(x)+2}.$$

(a) Find the inverse function $f^{-1}(x)$;

(b) What is the domain of $f^{-1}(x)$? (i.e. find the values of x for which $f^{-1}(x)$ is defined as a real function).

(c) Show that the equation

$$f^{-1}\left(\frac{\sin\theta}{4}\right) = 0$$

has no real solutions for θ.

4. Find the first *three* non-zero terms in the series expansion, in increasing integer powers of x, of the following functions:

$$(a)\ (1-x^2)^{1/3};\qquad (b)\ \frac{\exp(x)}{2-x};\qquad (c)\ \text{sech } x.$$

THE END