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M1M1: Progress Test 1: October 25th 2004

Write your name **clearly** on your answer book.

No calculators. No books/lecture notes.

50 minutes. Attempt all four questions.

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1. The function  $f(x)$  is defined as

$$f(x) = \frac{\cos x}{1 - \sin x}.$$

- (a) Is  $f(x)$  a periodic function? If so, what is its period?  
(b)  $f(x)$  can be written as the sum of an even function  $f_e(x)$  and an odd function  $f_o(x)$  so that

$$f(x) = f_e(x) + f_o(x).$$

Find  $f_e(x)$  and  $f_o(x)$ .

- (c) Show that

$$\frac{f_o(x)}{f_e(x)} = \sin x.$$

2. Put the following three rational functions in partial fraction form:

$$(a) \frac{x^2}{x^2 + 1}; \quad (b) \frac{x^2}{x^2 - 1}; \quad (c) \frac{1}{x^3 + 1}.$$

3. Define

$$f(x) = e^x + 2e^{2x} \quad \text{for all } x.$$

- (a) What is the range of  $f(x)$ ?  
(b) Find all real solutions  $\theta$  of the equation  $f(\log \theta) = 1$ .  
(c) Find the inverse function  $f^{-1}(x)$ .

4. In lectures, the functions  $\sinh x$  and  $\cosh x$  were defined as the odd and even parts of the function  $\exp(x)$ . We then also defined

$$\tanh x \equiv \frac{\sinh x}{\cosh x} \quad \text{and} \quad \operatorname{sech} x \equiv \frac{1}{\cosh x}.$$

- (a) Using the definitions, derive the identity

$$\cosh^2 x - \sinh^2 x = 1.$$

- (b) Using the result of part (a), or otherwise, derive an identity relating  $\tanh x$  and  $\operatorname{sech} x$ .

**THE END**