M1M1: Progress Test 1: October 31st 2003

Write your name and tutorial group number on your answer book.
No calculators. No books/lecture notes.
50 minutes. Attempt all four questions.

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1. The function $f(x)$ is defined as:

$$f(x) = \left(\frac{x+x^2}{x-x^2}\right)^{1/2}.$$

From lectures, it is known that this can be written

$$f(x) = f_e(x) + f_o(x)$$

where $f_e(x)$ is an even function of x and $f_o(x)$ is an odd function of x.

(a) Find explicit expressions for $f_e(\sin\theta)$ and $f_o(\sin\theta)$.

(b) Hence verify that

$$1 + [f_o(\sin\theta)]^2 = [f_e(\sin\theta)]^2.$$

2. Put the following two rational functions in partial fraction form:

$$(a)\ \frac{x^4}{x^2+1}; \qquad (b)\ \frac{1}{x^4+x^2+1}.$$

3. Find the first three non-zero terms in the series expansions (in non-negative powers of x) of the following functions:

$$(a)\ \frac{x^2+1}{x-1}; \quad (b)\ e^{\sin x}; \quad (c)\ \frac{1}{e^x+e^{-x}}.$$

4. Find the inverse function $f^{-1}(x)$ for the following two functions:

$$(a)\ f(x) = \frac{2x+5}{x-1};$$
$$(b)\ f(x) = \left(e^x - e^{-x}\right)^{1/2} \quad \text{for } x \geq 0.$$

THE END