
M1M1: Progress Test 1: October 28th 2002

Write your name and tutorial group number on your answer book.

No calculators. No books/lecture notes.

50 minutes. Attempt all four questions.

1. The function $f(x)$ is defined as:

$$f(x) = \left(\frac{1 - \sin 3x}{1 + \sin 3x} \right)^{1/2}.$$

- (a) Decompose $f(x)$ into the sum of an even function and an odd function.
(b) Is $f(x)$ periodic? If so, find its period.
2. Find the first three non-zero terms in the series expansions (in powers of x) of the following functions:

$$(a) (1 + x^3)^2; \quad (b) (1 + \exp(x))^3; \quad (c) \frac{1}{1 + \exp(x)}.$$

3. Calculate the following limits:

$$(a) \lim_{x \rightarrow 0} \left[\frac{\sin x^{1/2}}{1 - \cos x^{1/4}} \right]; \quad (b) \lim_{x \rightarrow 0} \left[\frac{\sinh x}{1 - e^x} \right];$$
$$(c) \lim_{x \rightarrow \infty} \left[\frac{1 + 3e^x + 17e^{2x}}{1 + 5e^x + 16e^{2x}} \right]; \quad (d) \lim_{x \rightarrow \infty} \left[\sqrt{e^x + 1} - \sqrt{e^x - 1} \right].$$

4. It was shown in lectures that the inverse function to $\sinh x$ (that is, $\sinh^{-1} x$) can be written in terms of the log function as

$$\sinh^{-1}(x) = \log \left(x + \sqrt{x^2 + 1} \right).$$

The function $\tanh x$ is defined as

$$\tanh x \equiv \frac{\sinh x}{\cosh x}.$$

In a similar way, the inverse function $\tanh^{-1}(x)$ can be written in terms of the log function. Find this function.

THE END