A1. Define the function $f(x)$ to be

$$f(x) = \frac{x^4+1}{x^4-1}.$$

(a) Sketch a graph of $f(x)$ carefully indicating on your sketch any important features.

(b) Sketch a graph of $f(e^x)$, again carefully indicating any important features.

A2. Find the first three non-zero terms in the series expansion, in powers of x, of the functions:

$$(a)\ e^{(1+x^3)^{1/2}}; \qquad (b)\ \log\Big(1+\log(1+x)\Big)$$

A3. (a) Calculate the value of the definite integral

$$\int_0^1 \frac{e^x}{e^{2x}+1}dx.$$

(b) Find the indefinite integral

$$\int \frac{\cosh 2x}{\sinh x}dx.$$

A4. (a) Find all complex solutions of the equation

$$(a)\ \tanh z = 3.$$

(b) Make a sketch in the complex plane of the solutions of the equation

$$|z-i| + |z+i| = 2\sqrt{2}.$$

A5. Find the solution of the differential equation

$$(a)\ \frac{dy}{dx} + \frac{y}{x^2} = e^{1/x}$$

which satisfies the condition $y(1) = 0$.

B1. (a) Find the derivative of $\log(1+\sqrt{x})$ from first principles.

(b) Suppose that a rectangle is to be inscribed inside an ellipse whose equation is given by

$$\frac{x^2}{4}+\frac{y^2}{9}=1$$

so that its four corners are each situated at some point on the ellipse. Assuming the sides of the rectangle are parallel to the axes, find the maximum possible area of the rectangle.

(c) By making use of the Leibniz rule, show that

$$\frac{e^x}{1-x}=\sum_{n=0}^{\infty}a_n x^n$$

where

$$a_n=\sum_{j=0}^{n}\frac{1}{(n-j)!}.$$

B2. (a) Find the indefinite integral

$$\int\frac{x^3+1}{x^3-1}dx.$$

(b) Define

$$I_n=\int_{\pi/4}^{\pi/2}\cot^n x dx.$$

Show that, for $n\geq 2$,

$$I_n=\frac{1}{n-1}-I_{n-2}.$$

(c) Find the solution of the ordinary differential equation

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}=x$$

satisfying the conditions that $y(0)=y'(0)=0$.