

**Question 4.** The proportions of  $N$  lectures during which Panopto works for M1M1, M1F, M1S and M1GLA are roughly

$$M = N \left[ \sqrt{N^2 + 1} - N \right], \quad F = \frac{(N^2 - 1) \tanh(N)}{2N^2 + \sqrt{N} + 1}$$

$$S = \log \left( \frac{N + 1}{N} \right) \log(\cosh(\frac{1}{2}N)), \quad G = \frac{N^2 \sin(1/N)}{2N + \tan^{-1}(N)}$$

Calculate the limits of these 4 expressions as  $N \rightarrow \infty$ .

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**Answer.**

$$M = N^2 \left[ (1 + N^{-2})^{1/2} - 1 \right] = N^2 \left( 1 + \frac{1}{2}N^{-2} + O(N^{-4}) - 1 \right) = \frac{1}{2} + O(N^{-2}) \rightarrow \frac{1}{2} \quad \text{(2 marks)}$$

$$F = \left( \frac{1 - N^{-2}}{2 + N^{-3/2} + N^{-2}} \right) \tanh N \rightarrow \frac{1}{2}(1) = \frac{1}{2} \quad \text{(2 marks)}$$

$$S \simeq \log(1 + N^{-1}) \log(\frac{1}{2}e^{N/2}) = (N^{-1} + O(N^{-2}))(\frac{1}{2}N - \log 2) = \frac{1}{2} + O(N^{-1}) \rightarrow \frac{1}{2} \quad \text{(2 marks)}$$

$$G = \frac{N^2(N^{-1} + O(N^{-3}))}{2N + \tan^{-1}(N)} \simeq \frac{N}{2N + \frac{1}{2}\pi} \rightarrow \frac{1}{2} \quad \text{(2 marks)}$$

For general clarity and mathematical style at marker's discretion (2 marks)

**Total : 10**

*[Notes for markers: Since all answers are  $\frac{1}{2}$  some may guess at least one of the answers – some sort of justification is required. Although I have designated 2 marks for general style you may to a large extent do as you choose, but of course you must be consistent across all the scripts. Remember they only have 15 minutes for this question. The students will eventually see a copy of this sheet. Don't forget to initial your work to help in the "Meet your Marker" sessions.]*