Question 1. The functions $f(x)$ and $g(x)$ are defined for all x by

$$f(x) = \frac{x+2}{x^2+x+1}, \qquad g(x) = f(x^2).$$

Find the even part of $f(x)$ and the even part of $g(x)$, which we write as $f_e(x)$ and $g_e(x)$. Express the difference $f_e(x) - g_e(x)$ in simplest form.

Answer. Clearly $g(-x) = g(x)$ and so $g(x)$ is even and so

$$g_e(x) = g(x) = \frac{x^2+2}{x^4+x^2+1}. \qquad \textbf{(1 mark)}$$

Now $f_e(x) = \frac{1}{2}(f(x) + f(-x))$ or **(1 mark)**

$$f_e(x) = \tfrac{1}{2}\left(\frac{x+2}{x^2+x+1} + \frac{2-x}{x^2-x+1}\right) = \tfrac{1}{2}\frac{(x+2)(x^2-x+1)+(2-x)((x^2+x+1)}{(x^2+x+1)(x^2-x+1)}.$$

$$= \frac{2x^2 - x^2 + 2 + 2x^2 - x^2 + 2 + \text{odd powers which must cancel}}{2[(x^2+1)^2 - x^2]}$$

$$f_e(x) = \frac{x^2+2}{x^4+x^2+1}. \qquad \textbf{(4 marks)}$$

Thus clearly $f_e(x) = g_e(x)$ and so $f_e - g_e = 0$. **(4 marks)**

Total 10

[Notes for markers: This question is mainly an algebaic exercise and may well be boring to mark - sorry! I didn't require them to express f_e in simplest form, but if they leave it seriously unsimplified you may deduct marks. I expect there will be several algebraic slips. If they make an early error, but follow it through competently, you may award partial credit. Remember that they have under 15 minutes for the question. In general you may redistribute marks between the chunks of 4 as you see fit. Indeed, you may, to a large extent, do as you choose, but of course you must be consistent across all the scripts. The students will eventually see a copy of this sheet.]

Addendum: I set this question by wondering whether there were any non-trivial functions such that $f_e(x) = f(x^2)$. Having found one, I thought I'd use it...