Name (IN CAPITAL LETTERS!): .... **TID:**

CID: .... **Personal tutor:** ....

Question 2.
Using the Mean Value Theorem (and NOT otherwise), find numbers A and B such that

$$A < \log(2.5) < B.$$

[Note: $B - A$ should be small, and you should use exact expressions rather than decimal approximations. Of course log means the natural logarithm.]

Answer. The MVT states that if a function $f(x)$ is continuous on $[a, b]$ and differentiable in (a, b), then

$$\frac{f(b) - f(a)}{b - a} = f'(\xi) \qquad \text{for some } \xi \in (a,\, b). \qquad \textbf{(1 marks)}$$

We choose $b = e \simeq 2.7$, $a = 2.5$ and $f(x) = \log(x)$ **(2 marks)**
so that $f'(\xi) = 1/\xi$. Then for some $e > \xi > 2.5$, we have

$$\log e - \log 2.5 = \frac{e - 2.5}{\xi}. \qquad \textbf{(2 marks)}$$

Now $\log(e) = 1$ and as x^{-1} is a decreasing function for positive x, we know that $0.4 = 1/2.5 > 1/\xi > 1/e$. Thus **(2 marks)**

$$\frac{e - 2.5}{2.5} > 1 - \log 2.5 > \frac{e - 2.5}{e} \quad \Longrightarrow \quad 2 - \tfrac{2}{5}e < \log 2.5 < \frac{2.5}{e} \qquad \textbf{(3 marks)}$$

Total 10

[Notes for markers: This could well be a messy question to mark – sorry! They may come up with other bounds – probably less accurate ones – for which you will have to check the arguments yourself. A common error will be confusing the bounds on ξ with the bounds on $f'(\xi)$. Some may not realise that $e > 2.5$. Be strict on anyone who gives a loose bound e.g. $B = 1000$, $A = -1000$, or who doesn't use the MVT. I personally would not insist they give the precise conditions for the MVT, but you may choose to. Remember that they have under 15 minutes for the question. You may, to a large extent, do as you choose, but of course you must be consistent across all the scripts. The students will see a copy of this sheet.]