Name (IN CAPITAL LETTERS!): .... **TID:** ☐

CID: .... **Personal tutor:** ....

Question 3.

The (independent) probabilities that the Panopto system works for lecturers A, E, J and M are respectively

$$A = \sin\left(3x^2\right), \qquad E = e^{-x}, \qquad J = 1 - \cos(x\sqrt{6}), \qquad M = \frac{1}{1+x},$$

where x is a positive real number, so small that x^6 can be neglected. Obtain series approximations for A, E, J and M. Which is more likely – that the system works for both A and E or for both J and M, and by how much? (In other words: which is bigger AE or JM, and by how much?)

Answer. A routine piece of series manipulation but requiring some accuracy.

$$A = (3x^2) + O(x^6) \qquad \textbf{(1 marks)}$$

$$E = 1 - x + \tfrac{1}{2}x^2 - \tfrac{1}{6}x^3 + \tfrac{1}{24}x^4 - \tfrac{1}{120}x^5 + O(x^6) \qquad \textbf{(1 marks)}$$

$$J = 6x^2/2 - 36x^4/24 + O(x^6) = 3x^2 - \tfrac{3}{2}x^4 + O(x^6) \qquad \textbf{(1 marks)}$$

$$M = 1 - x + x^2 - x^3 + x^4 - x^5 + O(x^6) \qquad \textbf{(1 marks)}$$

Thus neglecting terms of $O(x^6)$, we have

$$AE = 3x^2 - 3x^3 + \tfrac{3}{2}x^4 - \tfrac{1}{2}x^5 \qquad \textbf{(2 marks)}$$

Similarly,

$$JM = 3x^2 - \tfrac{3}{2}x^4 - x(3x^2 - \tfrac{3}{2}x^4) + x^2(3x^2) - x^3(3x^2) = 3x^2 - 3x^3 + \tfrac{3}{2}x^4 - \tfrac{3}{2}x^5$$

(2 marks)

Thus

$$AE - JM = x^5 + O(x^6) \qquad \textbf{(2 marks)}$$

Total 10

[Notes for markers: This week, the correct answers matter as well as the reasoning. This may well be a boring question to mark - sorry! I expect there will be several arithmetical slips in deriving the final answer. You may need to adjust the mark scheme for those who don't calculate AE and JM explicitly but go straight for the difference between the two. Remember that they have under 15 minutes for the question. You may, to a large extent, do as you choose, but of course you must be consistent across all the scripts. The students will see a copy of this sheet.]