

Name (IN CAPITAL LETTERS!): TID:

CID: Personal tutor:

Question 1.

The happiness, $h(x)$, of a Maths 1st year student on a Monday decreases first linearly until the test starts at $x = 0$, and then exponentially afterwards, so that approximately:

$$h(x) = \begin{cases} 1 - x & \text{for } x < 0 \\ e^{-x} & \text{for } x \geq 0 \end{cases} \quad (1)$$

- (a) What is the range of this function as x takes all real values?
- (b) Find the inverse function, h^{-1} over its appropriate domain.
- (c) If $h(x) = h_e(x) + h_o(x)$, where h_e is even and h_o is odd, then find h_o explicitly.

Answer. (a) By inspection, $h(x)$ can take all strictly positive values, so the range is $(0, \infty)$. Accept any sensible way of writing this, but deduct one mark if they don't exclude the value 0. **(2 marks)**

(b) Let $y = h(x)$. Then if $x < 0$, we have $x = 1 - y$ while if $x \geq 0$, we have $-x = \log y$. The value $x = 0$ corresponds to $y = 1$, so that the inverse is

$$h^{-1}(y) = \begin{cases} 1 - y & \text{for } y > 1 \\ -\log y & \text{for } 0 < y \leq 1 \end{cases} \quad (4 \text{ marks})$$

(c) The odd part of $h(x)$ is $[h(x) - h(-x)]/2$. If $x < 0$, then $-x > 0$, and so this is $(1 - x - e^x)/2$, whereas if $x > 0$, we have $(e^{-x} - 1 - x)/2$. So $h_o(x)$ is given by

$$h_o = \begin{cases} (1 - x - e^x)/2 & \text{for } x < 0 \\ (e^{-x} - 1 - x)/2 & \text{for } x \geq 0 \end{cases} \quad (4 \text{ marks})$$

This question is not too difficult, but care is needed with notation and dealing logically with the different ranges. There are other ways of writing the solutions, e.g. perhaps $h^{-1}(x)$ in (b,) but the answers should be defined clearly. **Total 10**

[Notes for markers: As ever, we are as interested in coherent reasoning as in the actual answers. There may be other ways of approaching the questions, whose worth you should assess yourself. Give credit for clarity of argument – you need not give full marks to a correct final answer which is not clearly explained. Likewise, you may, if you wish, penalise poor presentation. Remember that they have under 15 minutes for the question, though. You may, to a large extent, do as you choose, but of course you must be consistent across all the scripts. The students will eventually receive a copy of this sheet.]