Name (IN CAPITAL LETTERS!): .... **TID:** ☐

CID: .... **Personal tutor:** ....

Question 3.
At 9.00 this morning, I desperately asked my colleagues "I forgot to set my question. Quick – what are your favourite functions?" The M1F lecturer kept oscillating between two ideas, so I put him down for $\cos x$. M1GLA naturally chose a linear algebraic function, $1+x$, while M1S opted for the Normal distribution, $\exp(-x^2)$.
(i) Obtain the the power series expansion up to and including terms in x^5 for

$$f(x) = (\cos x)(1+x)\exp(-x^2).$$

(ii) If you substitute $x = \frac{1}{2}\pi$ in the quintic polynomial approximation, the answer is about 9.36, a long way from zero. Which of the following correctly describes why?

(a) Probably $\frac{1}{2}\pi > R$, where R is the radius of convergence of the infinite series.
(b) Although the cosine is approximately zero, the exponential is very large.
(c) Not enough terms have been included for the approximation to be good.
(d) There is no general formula for the solution of quintic equations.

Answer. (i) There are various methods of proceeding. We have

$$\begin{aligned}
&(1+x)(1-\tfrac{1}{2}x^2+\tfrac{1}{24}x^4+O(x^6))(1-x^2+\tfrac{1}{2}x^4+O(x^6))\\
&= (1+x)\left[1-\tfrac{3}{2}x^2+x^4(\tfrac{1}{2}+\tfrac{1}{2}+\tfrac{1}{24})+O(x^6)\right]\\
&= 1+x-\tfrac{3}{2}x^2-\tfrac{3}{2}x^3+\tfrac{25}{24}x^4+\tfrac{25}{24}x^5+O(x^6)
\end{aligned}$$

[For ease of marking, let's have 1 mark for each of the 6 terms. If you feel a single error has been unduly penalised, you may only deduct one mark for it. I expect the majority will not group the even function together and will have a harder calculation as a result.] **(6 marks)**

(ii) Each of the functions in the product have infinite series which converge for all x (they have been told). So the product is expected also to have infinite Radius of Convergence. Thus (a) is wrong, (b) is nonsense (the exponential is not large, it's less than 0.1), (d) is utterly irrelevant which leaves (c) as the only plausible explanation. We note that x^6 is not small when $x = \frac{1}{2}\pi \simeq 1.57$, and there is no evidence that the coefficients are getting small yet. So the answer is (c). **(2 marks)**

General layout, presentation and clarity of argument **(2 marks)**

Total 10