M1A1: Problem Sheet 6 Rotating Systems

1. A fairground ride involves a large hollow cylinder of radius $3m$ which can rotate about a vertical axis. People line the inner wall of the cylinder, which spins with angular velocity Ω. A person with mass $70kg$ has a coefficient of friction $\mu = 1$ with the wall. How great must Ω be so that the person remains at the same horizontal level when the floor is removed?

2. When a cylindrical cup of tea is stirred, assume all the tea rotates with a constant angular velocity $(0, 0, \omega)$. The tea surface is observed to adopt a curved shape, $z = f(R)$, where R is the distance from the axis of symmetry (the z-axis), and f is an unknown function.

In a rotating frame, consider the forces acting on a drop of tea in the free surface. If the tangent to the tea surface makes an angle ψ with the horizontal (so that $\tan\psi = f'(R)$), show that the effective gravity is perpendicular to the surface if

$$\tan\psi = \frac{\omega^2 R}{g} .$$

Deduce that the surface adopts the parabolic shape

$$z = z_0 + \frac{\omega^2 R^2}{2g} .$$

Is this why air bubbles on the tea surface "fall" to the centre of the cup?

3. A stone is dropped from the top of a tower of height h on the equator. Ignoring air resistance and terms quadratic in the Earth's rotation rate ω, show that when it hits the ground it has moved a distance d to the East, where

$$d^2 = \frac{8h^3\omega^2}{9g} .$$

Estimate d for a $50m$ tower.

4. In a frame rotating about the z-axis with angular velocity $(0, 0, \dot\theta)$, the apparent velocity and acceleration of a particle are $\mathbf{v}_{rot} = \dot r\,\widehat{\mathbf{r}}$ and $\mathbf{a}_{rot} = \ddot r\,\widehat{\mathbf{r}}$. Using the relation between $\mathbf{a}_{rot}$ and the real acceleration $\mathbf{a}_{in}$, show that as we previously found

$$\mathbf{a}_{in} = (\ddot r - r\dot\theta^2)\widehat{\mathbf{r}} + \frac{1}{r}\frac{d}{dt}(r^2\dot\theta)\widehat{\theta} .$$