M1A1: Problem Sheet 5 Gravitation & Orbits

(Throughout this sheet, m denotes an appropriate mass.)

1. It is proposed that the force between the Earth and Sun is actually

$$F(r) = \frac{GMm}{r^2}\left(1 + \frac{\delta}{r}\right) \qquad \text{for constant } \delta$$

where δ is much smaller than the distance from earth to sun. Calculate the orbital equation for $r(\theta)$ in this case. What is the main qualitative difference compared with the case $\delta = 0$?

2. Two masses m_1 and m_2 attract each other gravitationally. In lectures we showed that the mass m_1 orbits about the moving mass m_2 as if it were stationary with m_2 increased to $m_1 + m_2$. Show that an alternative description is that it behaves as if there were a stationary mass $m_2^3/(m_1 + m_2)^2$ at the centre of mass.

3. A satellite is in **circular** orbit around the earth. By considering the radial force balance, deduce Kepler's 3rd law.

How far must it be from the earth if it is **geo-stationary**, i.e. it remains above the same point on the earth's equator as the earth rotates?

4. A satellite in a low, circular orbit is subject to the slight air resistance $-mk\mathbf{v}$ where $\mathbf{v}$ is its velocity and $k > 0$ is constant. Show that its angular momentum decreases at fractional rate k, i.e. that $\dot{h} = -kh$.

If the change is sufficiently gradual that at all times the orbit can be regarded as circular (i.e. you can neglect $\dot{r}$ and $\ddot{r}$ in the equations), calculate the fractional rates at which it loses height and **gains** kinetic energy.

[Friction doesn't always slow you down! The satellite's **total** mechanical energy decreases exponentially, and its temperature rises accordingly due to the friction. There is thus a serious danger of it burning up as it enters the earth's atmsphere.]

5. The electrostatic force between two electric charges q_1 and q_2 separated by a distance r is

$$F_e = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} \qquad \text{where } \varepsilon_0 \text{ is constant.}$$

The force is attractive if q_1 and q_2 have opposite sign.

How can we certain that the force between the earth and sun is gravitational rather than electrostatic?