M1A1: Problem Sheet 2

Statics

1. A uniform cubical block of side $2a$ rests on a rough table with friction coefficient μ. A person tries to push the block, by exerting a force normal to one of the faces at a height h above the table where, $h > a$. As the force is slowly increased from zero, will the block begin to slide or will it begin to topple over an edge?
[Hint: assume friction is large enough for slipping not to occur, and consider the situation when the block just begins to topple over an edge.]

2. In lectures we considered a ladder resting on rough ground leaning against a smooth wall. Explain why it is always safe to climb half way along the ladder, if the ladder stands up at all.

3. External forces, $\mathbf{F}_i$ act at points $\mathbf{r}_i$ on a system of particles, for $i = 1 \ldots N$. Given that the forces sum to zero, as do their moments about the origin, show that the force moments about a general point, $\mathbf{R}$, sum to zero.

4. Three forces, $\mathbf{F}_0$, $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a rigid body at respectively the origin, $\mathbf{r}_1$ and $\mathbf{r}_2$. The body is in static equilibrium and $\mathbf{F}_1$ is not parallel to $\mathbf{F}_2$. Show that

$$\mathbf{F}_2 \cdot (\mathbf{r}_1 \wedge \mathbf{F}_1) = 0 \qquad \text{and} \quad \mathbf{F}_1 \cdot (\mathbf{r}_2 \wedge \mathbf{F}_2) = 0 \; .$$

Deduce that the five vectors must lie in a single plane and show that the lines of action of the three forces must pass through a single point.

5. An abseiler on an ice cliff attaches a rope of length L to the top of the cliff and to a harness on his body. When he is standing upright, the harness is a distance h above his feet, while his centre of mass is a distance d above his feet.
He attempts to brace himself with his feet against the smooth vertical cliff, with his body at a suitable angle.

Use Q4 to show that an equilibrium is possible. When must his feet be higher than his head?

[*No liability is accepted for any injury sustained in attempting to emulate any character in these questions.*

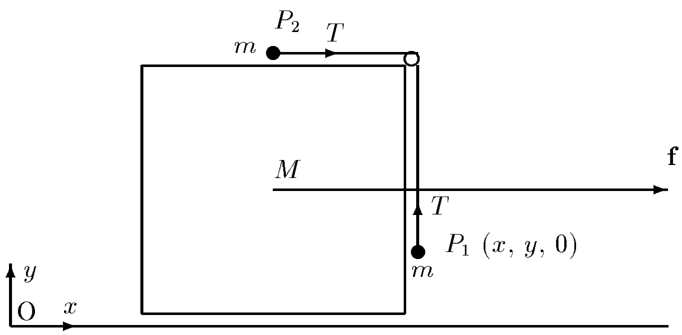
Is it sufficient for an equilibrium to exist for it to be "safe?" What other property should an equilibrium possess?]

(PTO)

The Sliding Block problem

6. *This was set as a coursework a few years ago:*

Two particles P_1 and P_2, each of mass m, are connected by a light inextensible string of length L. P_2 lies on a uniform cubical block of mass M and side a which rests on a table, while P_1 hangs over one side. All contacts are smooth, and P_1 remains in contact with the side of the block during the motion at position $(x, y, 0)$ as in the figure. A force $(f, 0, 0)$ is applied to the block in such a way that it moves in a straight line and does not rotate.



The following two statements are **false**. Explain the mistake, in each case:

(1) As P_2 is a horizontal distance $(L-(a-y))$ from the pulley at the corner of the block, it obeys the equation $T = -m\ddot{y}$, where T is the (unknown) tension in the string.

(2) A horizontal force balance on a combination of the block and P_1 implies $f = (m+M)\ddot{x}$.

Solve the problem correctly, find $\ddot{x}$ and show that

$$\ddot{y} = \frac{f - 2mg - Mg}{3m + 2M} \ .$$

[Note this means that P_1 can be lifted if f is big enough!]