UNIVERSITY OF LONDON

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

BSc/MSci EXAMINATION (MATHEMATICS) MAY 2002

This paper is also taken for the relevant examination for the Associateship

M1A1 Mechanics

DATE: 15th May 2002 TIME: 10:00 – 12:00 am

Credit will be given for all questions attempted, but extra credit will be given for complete or nearly complete answers.

Calculators may not be used.

 M1A1: 6 Pages

1. A space station consists of three identical modules of mass m at the vertices of an equilateral triangle of side a. The modules are held together by three taut cables. The triangle rotates with angular velocity $(0,\, 0,\, \omega)$ about the centre of mass, which is at rest at the origin. No external forces act on the system.

Calculate the speed of each module and the total kinetic energy K, momentum $\mathbf{P}$ and angular momentum $\mathbf{H}$ of the system.

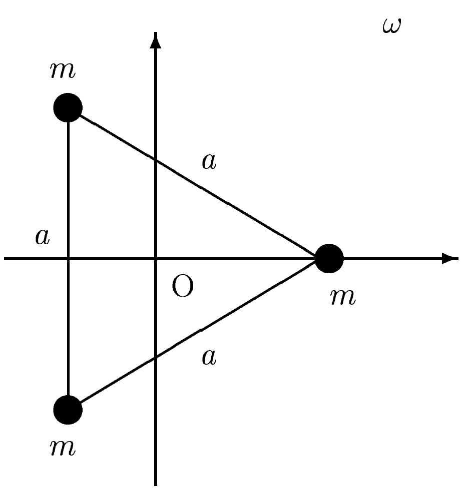
Find the tension T in each of the cables.

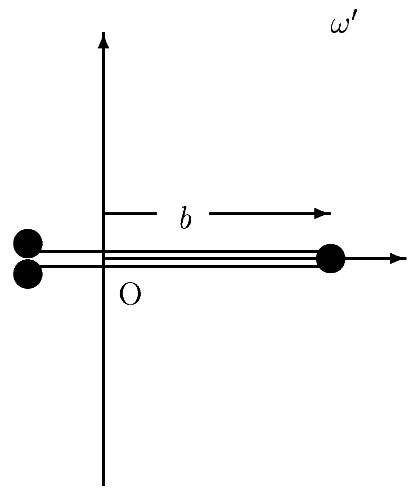
For repair work, two of the modules are brought alongside each other by slowly reducing the length of one of the cables using a winch mounted on one of the modules. After the operation, what are the new values K', $\mathbf{P}'$ and $\mathbf{H}'$ of the energy, momentum and angular momentum?

Relate b to a and show that the new rotation speed, ω' is given by

$$\omega' = \tfrac{3}{2}\omega \ ,$$

Discuss whether or not one should expect $K' - K = aT$.





2. A particle of mass m and electric charge q moves with velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$. Its equation of motion in an inertial frame is

$$m\frac{d\mathbf{v}}{dt} = q\,\mathbf{v} \wedge \mathbf{B}\ .$$

Show that the kinetic energy of the particle is constant during the motion.

Consider the magnetic field $\mathbf{B} = (0,\, 0,\, B)$, where B is constant. At time $t = 0$ the particle is at the origin with velocity $\mathbf{v} = (u_0,\, 0,\, w_0)$. Find the particle's position vector $\mathbf{r}(t) = (x(t),\, y(t),\, z(t))$ for all time.

Show that there is another inertial frame $(X,\, Y,\, Z)$ moving at constant velocity $\mathbf{U}$ relative to the first frame, in which the particle moves in a circle. Identify the relative velocity $\mathbf{U}$ and the centre and radius of this circle.

Turn over...

3. The position of a particle $x(t)$ obeys the equation

$$\ddot{x} + 2k\dot{x} + \omega_0^2 x = F_0 \cos \omega t \ .$$

State the physical dimensions of the positive constants k, F_0, ω and ω_0.

Find the general solution for $x(t)$.

Show that for large values of t the solution is independent of the initial conditions.

Calculate the amplitude of the motion at large time, and find the value of ω^2 for which this is maximum, assuming $\omega_0^2 > 2k^2$.

4. A particle of mass m moves in one-dimension under the influence of a smooth potential $V(x)$. State sufficient conditions for $x = x_0$ to be a stable equilibrium point.

If the particle is disturbed infinitesimally from such a point, show that it oscillates about the stable equilibrium, and calculate the frequency with which it does so.

A particle of mass $m = 1$ moves under the potential

$$V(x) = -\frac{\ln x}{x} \qquad \text{in} \quad x > 0\ .$$

Sketch the potential and classify the equilibrium points, if any.

At $t = 0$ the particle is at $x = 2$, and $\dot{x} = -u$, where $u > 0$. Describe the subsequent motion carefully, and identify any critical values of u.

Turn over...

5. A particle of mass m and electric charge q a distance r from the nucleus of an atom feels a force

$$F(r) = \frac{Aq}{r^2} \qquad \text{where } A \text{ is constant}$$

directed away from the atom if $Aq > 0$. No other force is relevant to the motion.

Starting from the equation of motion in polar coordinates (r, θ), show that the particle orbit satisfies

$$\frac{l}{r} = 1 + e\cos(\theta - \phi) \qquad \text{for constants } e \text{ and } \phi,$$

and relate the parameter l to the angular momentum per unit mass, h.

Express the kinetic energy of the particle as a function of θ, and defining a suitable potential for the force $F(r)$, deduce that the particle energy E is given by

$$E = \frac{A^2q^2}{2mh^2}(e^2 - 1) \ .$$