A **particle**, or **point mass**, is an object small enough to be considered to occupy a single point. It is defined by its (constant) mass, m, and a position vector $\mathbf{r}$ relative to an origin, O. Often we will refer to the **components** of the vector with respect to Cartesian axes, writing $\mathbf{r} = (x,\, y,\, z)$. The origin and axes are fairly arbitrary, and should be chosen for convenience (but they must form an **inertial frame** (see later). This means they must not accelerate.)

If the particle moves, its position vector will change with time. Its **velocity**, $\mathbf{v}$, is

$$\mathbf{v} \equiv \frac{d\mathbf{r}}{dt} \equiv \dot{\mathbf{r}} = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right) \equiv (\dot{x},\, \dot{y},\, \dot{z})\ .$$

If the velocity is zero, we say the particle is **at rest**, or **stationary**. Note that a particle at rest does not have to remain stationary for all time.

The **momentum** of a particle, $\mathbf{p}$, is defined as $\mathbf{p} = m\mathbf{v}$.

Similarly, the **acceleration** of the particle, $\mathbf{a}$, can be written

$$\mathbf{a} \equiv \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2} \equiv \ddot{\mathbf{r}} \equiv (\ddot{x},\, \ddot{y},\, \ddot{z})$$

Note that a dot above a letter represents a derivative with respect to time; two dots a double derivative. Be very careful when copying dots from the whiteboard!

The **displacement** or relative position of one particle with position vector $\mathbf{r}_1$ with respect to another, with position vector $\mathbf{r}_2$ is $(\mathbf{r}_1 - \mathbf{r}_2)$. Differentiating this, we obtain the **relative velocity** $(\mathbf{v}_1 - \mathbf{v}_2)$. It is a fundamental idea of Newtonian mechanics that motion is relative. Provided two particles have constant velocities it is equally valid to regard one as stationary and the other as moving, or vice versa.

It is important to familiarise yourself with vector notation and the rules of vector manipulation. For example, **NEVER** try to add a vector and a scalar. This is worse than being wrong – it is meaningless and tells the world you have no idea what you're writing.

The **scalar product** or **dot product** between two vectors is a convenient way of finding the **component** of one vector in the direction of another. For example, writing $\widehat{\mathbf{e}}_1 = (1,\, 0,\, 0)$, the velocity component in the x-direction is

$$\mathbf{v} \cdot \widehat{\mathbf{e}}_1 = \dot{x}\ .$$

More generally, the component of $\mathbf{F} = (F_1,\, F_2,\, F_3)$ in the direction of the **unit vector** $\widehat{\mathbf{n}} = (n_1,\, n_2,\, n_3)$ is

$$\mathbf{F} \cdot \widehat{\mathbf{n}} = F_1 n_1 + F_2 n_2 + F_3 n_3 = |\mathbf{F}||\widehat{\mathbf{n}}| \cos\theta = |\mathbf{F}| \cos\theta\ ,$$

where θ is the angle between the vectors $\mathbf{F}$ and $\widehat{\mathbf{n}}$.

Rigid bodies: A **rigid body** is a collection of particles which do not change their positions with respect to each other. In many cases, a rigid body can be treated the same as a particle with the same total mass at a special point known as the **Centre of Mass**. For example, the earth can be considered a particle while calculating its orbit around the sun. However, there is an important difference, in that a rigid body can **rotate**, as well as move with a given velocity. There may also be geometrical constraints – obviously a big body may bump into things which a particle would not notice.

Rotation about an axis: A rigid body can spin about an axis with a uniform **angular velocity**, so called because it is the rate of change of angles. It is convenient to define the angular velocity to be a vector, $\underline{\omega}$, pointing in the direction of the rotation axis. If the origin, O, lies on this axis and is at rest, then the velocity $\mathbf{v}$ of a particle at position $\mathbf{r}$ in the body, is given by the **vector product** or **cross product**

$$\mathbf{v} = \underline{\omega} \wedge \mathbf{r} \ .$$

The cross product of two vectors is a vector perpendicular to both, with magnitude

$$|\underline{\omega} \wedge \mathbf{r}| = |\underline{\omega}|\,|\mathbf{r}| \sin\theta \ ,$$

where θ is the angle between $\underline{\omega}$ and $\mathbf{r}$. Note that $|\mathbf{r}|\sin\theta$ is the distance from the particle to the rotation axis. The motion of a rigid body is completely described by the velocity, $\mathbf{v}_G$ of a particular point G, (usually the centre of mass) together with the angular velocity, $\underline{\omega}$ about an axis through G. Taking the origin at G, the velocity of a general point with position vector $\mathbf{r}_G$ is

$$\mathbf{v} = \mathbf{v}_G + \underline{\omega} \wedge \mathbf{r}_G \ .$$

Note that $\mathbf{v}_G$ and $\underline{\omega}$ may change with time. Fully three-dimensional rotation is complicated and will not be covered in this course.

Physical dimensions: There are three basic quantities of mechanics: Mass [M], length [L] and time [T], which we usually measure in kilograms, meters and seconds. For an equation to make sense, every term in it must have the same dimensions. This is often a useful check on algebra. Here is a list of the dimensions of various important mechanical quantities we shall meet in this course:

Mass	[M]	Displacement	[L]	Time	[T]
Velocity	[L/T]	Angular Velocity	[1/T]	Momentum	[ML/T]
Force	[ML/T^2]	Acceleration	[L/T^2]	Energy or Work	[ML^2/T^2]
Moment (Torque)	[ML^2/T^2]	Angular Momentum	[ML^2/T]	Power (work rate)	[ML^2/T^3]

Of these, mass, time, energy and power are **scalars**; the rest are **vectors**. The magnitude of **displacement** is called **distance** and the magnitude of **velocity** should strictly be called a **speed.** Unfortunately, sometimes these terms are used incorrectly. For example, when people talk about the 'velocity of light' they usually mean 'speed of light.'

Note also that **weight** is really a **force**. When we say someone weighs 60kg, we really mean their **mass** is 60kg, and their weight is the gravitational force which acts upon it.