On this sheet we discuss the various kinds of forces which one body can exert on another. First we consider forces which act at a distance – gravity and electromagnetism.

Gravity: It was observed by Galileo that bodies of different masses falling to earth accelerate with the same constant acceleration, $\mathbf{g}$. We infer that a gravitational force

$$\mathbf{W} = m\mathbf{g} \qquad (\textbf{weight}) \tag{1.12}$$

acts on a body of mass m. Note $\mathbf{g}$ can be treated as constant for systems on the earth's surface, but it varies over large distances, as we'll see later.

Electromagnetism: A particle with **electric charge** q travelling with velocity $\mathbf{v}$ in an electric field $\mathbf{E}$ and a magnetic field $\mathbf{B}$ feels a force

$$\mathbf{F}_e = q(\mathbf{E} + \mathbf{v} \wedge \mathbf{B}) \ . \tag{1.13}$$

Two charged particles interact through the electromagnetic field they generate. You do not need to remember this formula.

Contact forces: A rigid body in contact with another body prevents it from occupying its space by exerting a force. If the contact is **smooth**, this force is perpendicular to the tangent plane of contact, and is called the **normal reaction.** It acts in the direction to separate the bodies, and does not inhibit motion in the tangent plane.

Friction: If the body surfaces are **rough**, a **frictional force** may act in this plane. Such a force acts to inhibit or slow down relative motion of the contacting bodies. The maximum amount of friction which can act is proportional to the magnitude of the normal reaction. Thus if $\mathbf{N}$ is the normal reaction and $\mathbf{D}$ the frictional force

$$|\mathbf{D}| \leqslant \mu|\mathbf{N}| \ , \qquad \text{where} \quad \mu \text{ is the (constant) } \textbf{friction coefficient.} \tag{1.14}$$

The value of μ depends on the surfaces of the bodies in question. Sometimes μ differs according to whether relative motion occurs or not, and then we must define coefficients of **static friction** and **dynamic friction**. We shall neglect this effect.

Strings and springs: Two bodies may be connected together by some device. For example, an **inextensible string** exerts a constant **tension** along its length. This tension can only be directed inwards from the ends along the string; otherwise the string goes slack. A **rod** can support both tension and **compression**, so that essentially the tension can be negative.

An **elastic string** can be stretched, in which case it exerts a force **proportional to its extension**. Again, it can only be pulled outwards. A **spring** is an elastic string which can also be compressed. If a spring of natural length a lies along the x-axis for $0 < x < a$, and the end at $x = a$ is extended to $x = a + s$, it exerts a **restoring force S**

$$\mathbf{S} = (-ks, 0, 0) \ , \qquad \text{where } k \text{ is the spring constant.} \tag{1.15}$$

The same equation holds for an elastic string provided it is stretched rather than compressed.

Strings and springs are usually considered to be light enough that their mass can be neglected. If they do have mass, then this must be taken into account.

A **pulley** is a device for supporting strings while allowing them to slide freely. If a string is in contact with a rough body, its tension may decrease dramatically because of friction. This is why a few turns of a rope around a capstan can tether a ship.

When a body is called **uniform** this usually means constant **density**, so that the centre of mass is in the middle, where you would expect.

For a body with **continuously varying** density $\rho(\mathbf{x})$ in a volume V, the generalisation of equation (1.3) defining the centre of mass vector is

$$\mathbf{R} = \frac{1}{M}\int_V \rho(\mathbf{x})\mathbf{x}\,dV \qquad \text{where} \quad M = \int_V \rho(\mathbf{x})dV \ . \tag{1.3'}$$

We can generalise similarly the definition of, for example, momentum.

Solving mechanical systems

We have now defined a set of building blocks which we can use to build up quite complex mechanical systems.

Example problem 1: A rigid body B_1 of mass m_1 lies on a smooth table which is fixed to the floor. The body is connected by a light inextensible string to a body B_2 of mass m_2 which hangs vertically over the end. The contact between the string and the table is smooth. What is the acceleration of the mass m_2?

The strategy for solving such problems is as follows:

(a) Split the system up into suitable parts. Choose suitable variables or coordinates to represent the possible motion. Use intelligence!

(b) For each part, define the forces, accelerations and velocities which act. For complex systems, use separate diagrams for each. Write down Newton's laws for each part, being sure that you are in an inertial frame. This may involve one, two or three force components, and may involve a torque (moment) balance.

(c) Consider any extra constraints, e.g. length of string being fixed, coefficients of friction, zero relative velocities if no slipping occurs.

(d) Eliminate unwanted variables, and solve the problem.

Statics

If no motion occurs, Newton's Laws for a system of particles reduce to $\mathbf{F} = 0$ and $\mathbf{G} = 0$. Usually such problems involve rigid bodies.

Example problem 2: A uniform ladder of mass m and length $2L$ rests against a vertical smooth wall making an angle α with the ground. A man of mass M slowly climbs up the ladder. If the coefficient of friction between ladder and ground is μ, determine whether or not he can climb safely to the top.