Civil Eng. 2 Mathematics **2009** **Problem sheet 3 Fourier Series**

This sheet can be found on the Web: http://www.ma.ic.ac.uk/~ajm8/MEng26

1. Find the Fourier series of the function $f(x)$ where

$$f(x) = 0 \qquad \text{for} \qquad -\pi < x < 0 \qquad \text{and} \quad f(x) = 1 \qquad \text{for} \quad 0 < x < \pi \ .$$

What is the value of the series at $x = 0$, where $f(x)$ is discontinuous?

{*Answer:* $f(x) = \frac{1}{2} + \frac{2}{\pi} \sum_{\text{n odd}} \sin nx/n$}

2. Show that if $-\pi < x < \pi$,

$$x^2 = \tfrac{1}{3}\pi^2 + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx \ ,$$

By differentiating this series, infer the Fourier series of x in the same interval. By integrating the series, and using the series for x you have just found, find a similar series for x^3.

{*Answer:* $x^3 = \sum (-1)^n \left[\frac{12}{n^3} - \frac{2\pi^2}{n}\right] \sin nx$}

3. Using Parseval's theorem for the series for x, x^2 and x^3 calculated in question 2, show that

$$(a) \ \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \ , \qquad (b) \ \sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90} \ , \qquad (c) \ \sum_{n=1}^{\infty} \frac{1}{n^6} = \frac{\pi^6}{945} \ .$$

4. Show that the **half-range sine series** for $f(x) = 1 + x/L$ in $0 < x < L$ is

$$\sum_{n=1}^{\infty} \frac{2}{n\pi} \left[1 - 2(-1)^n\right] \sin\left(\frac{n\pi x}{L}\right) .$$

Sketch the function represented by the series in the range $-2L < x < 2L$.