# BioFluids Lecture 9-10: Flights of fancy: the birds and the bees

Last time we concluded that efficient swimmers were using their caudal fin, often in the form of a lunate tail, as wings to generate hydrodynamical lift. Today we discuss mechanisms of lift generation and flight.

We will assume the wings are thin with total wing area $W$ and length $2b$. Their shapes are streamlined so that the boundary layers remain attached up to the trailing edge. As in lecture 2, experiment shows that the profile drag, $D_P$ and lift, $G$, can be written

$$D_P = \tfrac{1}{2} C_D \rho S U^2, \qquad G = \tfrac{1}{2} C_L \rho W U^2, \tag{9.1}$$

where $S$ is the area from the front and $W$ is the wing area from on top. The drag and lift coefficients, $C_D$ and $C_L$ are determined by the geometry and Reynolds number. We begin by considering steady flight.

**2-D steady flight:** The theory for steady, 2-D lift is not hard to understand. We can solve for potential flow around 2-D bodies of any shape using conformal mapping techniques. A typical aerofoil shape has a pointed trailing edge, at which the potential flow speed may be infinite, with a corresponding very low pressure. The vorticity-laden boundary layer is unable to cope with the associated large adverse pressure gradient, and it separates, carrying a vortex away from the wing. Circulation, $\Gamma$, of an opposite sense to the shed vortex remains around the wing, and if it is of suitable magnitude it can ensure that the potential flow is finite at the trailing edge. This is known as the Kutte condition. For example, consider a flat plate from $x = -a$ to $x = a$ in a flow of speed $U$. The mapping

$$z = \zeta + \frac{a^2}{\zeta}, \tag{9.2}$$

where $z = x + iy$, takes the flat plate to the circle $|\zeta| = a$ in the $\zeta$-plane. If we have a uniform flow inclined at an angle $\alpha$, the complex potential around the circular cylinder is

$$w(\zeta) = U\left(e^{-i\alpha}\zeta + \frac{a^2 e^{i\alpha}}{\zeta}\right) - \frac{i\Gamma}{2\pi}\log\zeta, \tag{9.3}$$

The velocity about the flat plate is

$$u - iv = \frac{dw}{dz} = \frac{dw}{d\zeta}\frac{d\zeta}{dz} = \left[U\left(e^{-i\alpha} - \frac{a^2 e^{i\alpha}}{\zeta^2}\right) - \frac{i\Gamma}{2\pi\zeta}\right]\frac{\zeta^2}{\zeta^2 - a^2}. \tag{9.4}$$

The velocity at the trailing edge $\zeta = a$ can be made finite by choosing

$$\Gamma = -4\pi U a \sin\alpha. \tag{9.5}$$

This then gives a lift force

$$G = 4\pi\rho a U^2 \sin\alpha, \tag{9.6}$$

perpendicular to the incoming flow velocity. The flat plate flow still has a singularity at the leading edge ($\zeta = -a$). This can be avoided by considering circles in the $\zeta$-plane which enclose $\zeta = -a$ but still pass through $\zeta = a$. These map onto shapes in the $z$-plane called Joukowski aerofoils, which are very convenient analytic wing shapes.

The lift expression (9.6) is valid so long as the boundary layer remains attached, which it does for small angle of attack, say $\alpha \leqslant 15^\circ$. Higher values of $\alpha$ lead to *stall* and a massive increase of drag. This is generally avoided when flying, although pigeons deliberately stall when landing.

**3-D steady flight:** Once the starting vortex has been shed from a 2-D aerofoil, it can be forgotten about. The remaining flow is potential, as the circulation is about an apparent vortex line inside the wing, which extends to infinity. When the wing has a finite extent in the 3rd dimension, however, the situation is more complex. Vortex lines cannot stop abruptly and must close or head off to infinity. A 3-D potential flow can provide no lift force, so during flight vorticity must continually be shed into the wake, and it must have a streamwise component. This can be calculated using Prandtl's 'lifting-line theory.'

We assume a thin wing cross-section which varies slowly in the $z$-direction, so that $a = a(z)$, we might suppose that the circulation at point $z = z_1$ will be some function $\Gamma(z_1)$, which is given by (9.6) with some effective angle of attack $\alpha(z_1)$. Then by Kelvin's circulation theorem, we expect streamwise vorticity $-d\Gamma/dz$ to be shed into the wake. We can then sum the effects of all the semi-infinite lines of shed vorticity in the wake to provide the downward velocity at the leading edge as the principle value integral

$$v(z) = -\frac{1}{4\pi}\int_{-b}^{b} \frac{\Gamma'(z_1)}{z - z_1}\, dz_1. \tag{9.7}$$

This additional velocity alters the effective angle of attack. Putting all this together with (9.5) leads to an integral- equation for $\Gamma$. Once this is solved, the total lift can be found by integrating along the wing. Details can be found in Childress (1981) or Batchelor (1967). As the effective velocity (including the effect of the wake) at the leading edge is not parallel to the oncoming flow, there is a component of the 'lift' in the downstream direction proportional to the integral of $v\Gamma$ along the wing. This increase in the drag force, known as the induced drag, is written

$$D_I = \frac{kG^2}{\frac{1}{2}\rho U^2 b^2}, \tag{9.8}$$

where $k$ is a dimensionless constant equal to $1/\pi$ for an elliptic wing profile, and which doesn't vary much from this value.

**Steady Horizontal Flight:** Suppose our bird exerts a mean thrust $T$. Then for steady horizontal flight the lift must balance the weight, $G = mg$, and we have the simple force balance

$$T = D = D_P + D_I = \tfrac{1}{2}C_D\rho S U^2 + \frac{k(mg)^2}{\frac{1}{2}\rho b^2}\frac{1}{U^2}. \tag{9.9}$$

Viewed as a function of $U$, the drag has a minimum at the value $U = U_m$ when the profile drag equals the induced drag, where

$$U_m = \left[\frac{4k(mg)^2}{C_D\rho^2 b^2 S}\right]^{1/4}. \tag{9.10}$$

At this optimal flight speed, the bird can fly a given distance for minimal work. If instead we want to find the speed which permits the longest time airborne for given work expenditure, we should consider the power, $TU$, and minimise that, whch will give a different optimising speed. However, flight actually at $U = U_m$ would not be stable. A slight decrease in the flight speed would lead to an increased drag and hence a further decrease in flying speed. So for steady stable flight we expect a speed slightly above $U_m$, so that a decrease in $U$ leads to a decrease in $D$ and hence a restoring acceleration.

**Gliding:** Now suppose the bird exerts no thrust $T$, and so begins to descend. While we might have doubts about ignoring unsteady effects during flapping flight, the bird can clearly adopt this steady configuration. It can glide at a downwards angle $\theta$ to the horizontal if

$$G = mg\cos\theta, \qquad D = mg\sin\theta \quad \Longrightarrow \quad \tan\theta = D/G. \tag{9.11}$$

The glide angle $\theta$ affording minimum energy loss is again attained at $U = U_m$. If the bird wishes to glide at a slower speed (smaller $\theta$) then it should increase $b^2S$ which means extending its wings. In contrast if it wishes to dive, it pulls in its wing tips. Hunting birds often glide at zero velocity relative to the ground near cliffs where an appropriate upward wind can be found.

**Flying for Free:** Under suitable circumstances, a bird may extract energy from variations in the wind strength and direction. The simplest way of doing this is to find a *thermal*, a vertically rising air column. Within this thermal, it glides downwards relative to the air, while banking (altering its inclination to the vertical), which causes it to follow a helical path within the extent of the thermal. If the updraft in the thermal is greater than the vertical descent relative to the air, the bird will gain height. From its new vantage point it can leave the thermal, gliding downwards until it finds a new thermal and repeat the process. Vultures and migrating birds are observed to do this.

**Soaring:** In the absence of land topography, more sophisticated manoeuvres are used by the albatross to extract energy from a horizontal shear. Suppose the wind is a 1-D shear, $\mathbf{u} = (U(z),\, 0,\, 0)$, where $z$ is the vertical coordinate. Let a bird flying in this shear be at position $(X(t),\, Y(t),\, Z(t))$ with velocity $(u,\, v,\, w)$ relative to the wind. We work in a non-inertial frame travelling with the air velocity at the instantaneous position of the bird. This frame has the acceleration in the $x$-direction

$$U'(Z)\frac{dZ}{dt} = wU'. \tag{9.12}$$

If we apply Newton's laws in such an accelerating frame, we must include a fictitious force $-mwU'$ in the equation of motion of the bird. This force does work at a rate $-muwU'$. All the bird need do is make sure that this is on average positive and it can extract energy from the mean flow. In other words, if $U' > 0$, it must arrange that $\overline{uw} < 0$. The albatross arranges to fly upwind ($u < 0$) when going up ($w > 0$) and downwind when going down. This soaring motion is very efficient. There are similarities with the way turbulent fluctuating velocities extract energy from the mean flow – $\overline{uw}$ can be thought of as a Reynolds stress.

## Why can't we fly?

The largest flying birds are about 12kg. We can easily understand why there is an upper limit for bird mass using **allometric** arguments. If the lift $G$ balances the body mass, an animal of characteristic length-scale $L$ must generate a lift $G \propto L^2U^2 \propto mg \propto L^3$. Thus the speed must scale as $L^{1/2}$ It follows, that the rate at which birds must do work as they fly, $TU$, scales as $L^{7/2}$. Now as we shall see in a later lecture, the metabolic rate of all animals scale very closely as $L^{9/4}$. It follows that for small enough animals the metabolic rate will be lower than the power needed to fly, but that as $L$ increases, eventually the two curves will cross, and the power needed will be less than that available by the standard chemical processes in the body. This shows an upper mass limit must exist for animals of similar design.

## Who flies better: the birds or the bees?

*When thou seest an eagle, thou seest a portion of genius; lift up thy head!* – William Blake

But: *Aquila non capit muscas. (An eagle does not catch flies.)* – Latin proverb

Each of the main categories of fliers have developed ways of optimising flight. Bird feathers have evolved to equalise the wing loading, and their permeability helps control the boundary layers. Their muscles also work more efficiently than those of the insects. In contrast, insects with their exoskeleton and its capacity for elastic oscillations are very well adapted for very rapid wing oscillations up to 500Hz. Insect flight might therefore be expected to be more time-dependent. Extra complexities are introduced by unsteady flight. The frequency parameter (Strouhal number) we defined in lecture 2 is important here. If $St \equiv \omega a/U < 1$, where $\omega$ is the frequency of periodic motions, then the wing can shed vorticity at a high enough rate to maintain the Kutte condition. In practice $St \simeq 0.3$ for almost all insects and birds, which is unlikely to be a coincidence. Animals have been flying for about $3 \times 10^8$ years, and are really good at it. In contrast, the mechanisms for understanding certain forms of insect flight have been understood for about $3 \times 10^1$ years, or even less.

Unsteady bird flight can be modelled using the sort of techniques we used for fish swimming, extended for large amplitude motions. The geometry is more 3-dimensional and the shed vortex sheet is wavy. The wing shape and inclination differ during the up and down phases of flapping flight, because of the differing effective angle of attack.

One major difficulty with insect flight when viewed from a classical aerodynamic perspective, is that even at low Strouhal number, the shed vorticity may remain too close to enable optimal wing performance. This is known as the Wagner effect. A significant advance in understanding insect flight was made by Weis-Fogh (1973) who proposed **the clap and fling** mechanism, which we now analyse.

## The clap and fling mechanism for insect flight

Kelvin's circulation theorem ensures that if viscous forces can be neglected then the circulation around a 2-D body remains zero if it is zero initially. However, if the single body ruptures into two, each part may retain sizable, but oppositely signed, circulation. In the clap and fling mechanism, at the start of each flapping cycle the wings are clapped together, possibly emitting a propulsive jet, and clearing any residual circulation from the previous cycle. They are then peeled apart entraining fluid into the widening gap. When the angle between the two wings is about 120°, the wings are flung apart, inducing circulation around each wing. They are then moved forward generating substantial lift, and shedding vorticity as the 2-D approximation breaks down. This cycle is repeated many times per second.

The irrotational 2-D part of the motion can be found using conformal mappings. The induced circulation follows without need of the Kutta condition. We let the $x$-axis in the complex $z$-plane be the symmetry line between two wings joined at the origin. One wing extends from the origin to $z = ae^{i\alpha}$. The angle $\alpha = \Omega t$ as the wings move apart each with angular speed $\Omega$. The upper half $z$-plane, with the rotating wing, can be transformed to the upper half $Z$-plane using the Schwarz-Christoffel mapping given by

$$\frac{dz}{dZ} = K \left( \frac{Z-1}{Z+1} \right)^{\alpha/\pi} \frac{Z-c}{Z-1}, \tag{9.13}$$

for suitable constants $K$ and $c$. Equivalently, we can write

$$ze^{-i\alpha} = Kf(Z) \qquad \text{where} \quad f(Z) = \int_{-1}^{Z} \left( \frac{1-Z}{Z+1} \right)^{\alpha/\pi} \frac{c-Z}{1-Z}\, dZ, \tag{9.14}$$

which we can relate to the beta-function. We want both $Z = 1$ and $Z = -1$ to correspond to the origin $z = 0$, which fixes $c$. We find that

$$c = 1 - \frac{2\alpha}{\pi} \qquad \text{and} \quad K = \frac{a}{f(c)}, \tag{9.15}$$

since $K$ is fixed by $z = ae^{i\alpha}$ at $Z = c$, so that $K = a/f(c)$ from (9.14).

Now the normal velocity is known on the real $Z$-axis. If the complex potential is $\omega$, then its imaginary part is the streamfunction $\psi$. On the symmetry line in the $z$-plane $\psi = 0$, while at a distance $r$ along the wing, $z = re^{i\alpha}$ and $\psi = -\frac{1}{2}\Omega r^2$, where $\Omega$ is its angular speed. Thus the complex potential must satisfy on the real $Z$-axis,

$$\Im m(\omega) = \begin{cases} -\frac{1}{2}\Omega K^2 [f(Z)]^2 & \text{for } \; -1 < Z < 1 \\ 0 & \text{for } Z < -1 \text{ and } Z > 1. \end{cases} \tag{9.16}$$

This enables us to write down the complex potential in the form

$$\omega(Z) \equiv \phi + i\psi = \frac{\Omega K^2}{2\pi} \int_{-1}^{1} \frac{[f(\zeta)]^2}{Z-\zeta}\, d\zeta, \tag{9.17}$$

since we then have an analytic function which is real on the real $Z$-axis, except where the integrand has a singularity where we pick up $\pi i$ times the residue at $\zeta = Z$. The flow can now be evaluated. In particular we can calculate the circulation around each wing. This is easiest to evaluate as the change in $\phi \equiv \Re e(\omega)$ as we perform a circuit of the wing, which is equivalent to moving from $Z = -1$ to $Z = +1$ in the $Z$-plane. Thus the circulation $\Gamma$ is given in terms of $\omega(Z)$ as

$$\Gamma = \omega(1) - \omega(-1) = \frac{\Omega K^2}{2\pi} \int_{-1}^{1} [f(\zeta)]^2 \left[ \frac{1}{1-\zeta} + \frac{1}{1+\zeta} \right] d\zeta = \frac{\Omega K^2}{\pi} \int_{-1}^{1} \frac{[f(\zeta)]^2}{1-\zeta^2} \, d\zeta. \quad (9.18)$$

We note that $f(\pm 1) = 0$, so that this expression is finite. We can now evaluate this expression for any angle $\alpha$. We find it varies only slightly with $\alpha$ except near $\alpha = 0, \pi$, so that it is not critical precisely when the wings separate. This also indicates that not much force is required to separate the wings, for the pressure difference between the inside and the outside is, by Bernoulli's theorem,

$$[p] = \left[ -\rho \frac{\partial \phi}{\partial t} \right] = -\rho \frac{d}{dt} [\phi] = -\rho \frac{d\Gamma}{dt} \ll \rho \Omega^2 a^2. \quad (9.19)$$

The theory can be extended to allow for viscous effects (not so negligible for insects) and for the three-dimensional vortex shedding. The conclusion is that the clap and fling mechanism permits rapid generation of lift.

Nevertheless, not all flying insects use it, preferring to establish a **leading edge vortex**. We have already alluded to the possibility of separation at the leading edge at a high angle of attack, which is often a precursor to undesirable stall. Nevertheless, many aircraft designs, such as the delta-wing, encourage such separation, as the low pressure in the separated region enhances lift strongly. If this separated flow can be encouraged to reattach to the wing downstream, the expected large increase in profile drag can be avoided. This is what happens in subsonic delta-wing flight at a moderately high angle of attack, a process known as 'dynamic stall'. (At supersonic speeds the swept wing is designed mainly to avoid the shock wave from the aircraft nose; superb fliers though they be, insects do not break the sound barrier!) Essentially, the oncoming flow has a sideways component on a swept wing which carries the vortex off the tip of the wing.

Recent experiments with both live and robotic insects have indicated that insect flight is often associated with spiral leading edge vortices. A pressure gradient is induced along the wing partly by the centrifugal effect of flapping, and partly by the increase in vortex size along the wing.

Finally, we should mention schooling of fish and formation flight. When travelling in groups, birds and fish naturally position themselves in hydrodynamically efficient configurations. Typically, adjacent fish move their caudal fins out of phase with their neighbours to the side, with the following row offset to make use of the shed vorticity. Similar vortical considerations lead to the 'V'-formation of flocks of birds.