Central Clearing Valuation Adjustment

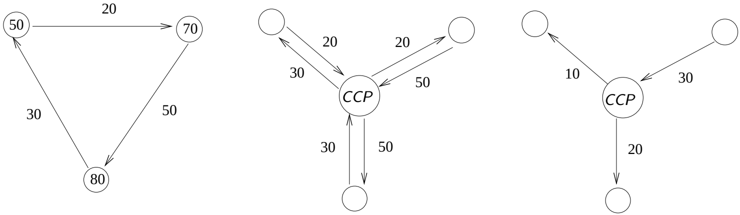
Stéphane Crépey (joint work with Yannick Armenti, arXiv 2015)
Université d'Evry Val-d'Essonne, Laboratoire de Mathématiques et Modélisation d'Evry (LaMME)

2nd London Paris Bachelier Workshop
25-26 September 2015, Kings College London.

This research benefited from the support of LCH.Clearnet in Paris and of the "Chair Markets in Transition" under the aegis of Louis Bachelier laboratory, a joint initiative of École polytechnique, Université d'Évry Val d'Essonne and Fédération Bancaire Française.

Introduction

- Central clearing is becoming mandatory for a vast majority of products



- Variation and initial margins versus mutualized default fund
- Supposed to eliminate counterparty risk, but at the cost for members of funding all the margins
- In this work we study the cost of the clearance framework for a member of a clearinghouse

  CCVA central clearing valuation adjustment

Outline

- We model a service of a clearinghouse dedicated to proprietary trading (typically on a given market) between its members, labeled by $i \in N = \{0, \ldots, n\}$
- The portfolio of any member is assumed fixed (unless it defaults)
- In practice, transactions with defaulted members are typically reallocated through a gradual liquidation of assets in the market (see Avellaneda and Cont (2013)) and/or through auctions among the surviving members for the residual assets at the end of the liquidation period
- For ease of analysis in this work, we simply assume the existence of a risk-free buffer that is used by the clearinghouse for replacing defaulted members in their transactions with others after a period of length δ

Member i's Portfolio Mark-to-Market Pricing Formula

- $\beta_t P^i_t = \mathbb{E}_t \left(\int_t^{\bar{T}} \beta_s dD^i_s \right)$, $t \in [0, \bar{T}]$

$\mathbb{E}_t$ conditional expectation given $(\mathcal{G}_t, \mathbb{Q})$

$\beta_t = e^{-\int_0^t r_s ds}$ risk-neutral discount factor at the OIS rate process r_t

  - the best market proxy for a risk-free rate
  - reference rate for the remuneration of the collateral

D^i contractual dividends

  - viewed from the perspective of the clearinghouse
  - $+1$ means 1 paid by the member i

$\bar{T}$ a time horizon relevant for the clearinghouse

  - if there is some residual value in the portfolio at that time, it is treated as a terminal dividend $(D^i_{\bar{T}} - D^i_{\bar{T}-})$

- But, ignored by the above mark-to-market pricing formula, any member i is defaultable, with default time τ_i and survival indicator process $J^i = \mathbb{1}_{[0,\tau_i)}$

Breaches

- For every time $t \geq 0$, let $t^{\delta} = t + \delta$ and let $\widehat{t}$ denote the greatest $lh \leq t$.
- For each member i, we write

  $C^i = VM^i + IM^i + DF^i$

  $Q^i_t = P^i_t + \Delta^i_t$ with $\Delta^i_t = \int_{[\tau_i, t]} e^{\int_s^t r_u du} dD^i_s$, $\chi_i = (Q^i_{\tau_i^{\delta}} - C^i_{\widehat{\tau}_i})^+$,

  $\xi_i = (1 - R_i)\chi_i$ where R_i denotes a related recovery rate

  - $R_i = 0$ modulo DVA / DVA2 issues

- For $Z \subseteq N$, let $\tau_Z \in \mathbb{R}_+ \cup \{\infty\}$ denote the time of joint default of names in Z and only in Z.
  - Joint defaults, which can be viewed as a form of "instantaneous contagion", is the way we will model credit dependence between members.

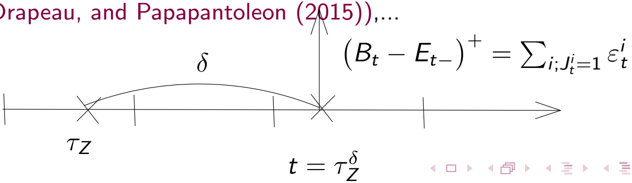
Lemma

At each liquidation time $t = \tau_Z^\delta = \tau_Z + \delta$ such that $\tau_Z < \bar{T}$, the realized breach for the clearinghouse (residual cost after the margins of the defaulted members have been used) is given by

$$B_t = \sum_{i \in Z} \xi_i$$

Equity and Unfunded Default Fund

- Equity (skin-in-the-game of the CCP) $E_{IY} = E^{\star}_{IY}$ and, at each $t = \tau_Z^{\delta}$ with $\tau_Z < \bar{T}$,
$$\Delta E_t = -(B_t \wedge E_{t-}).$$
- As in a senior CDO tranche, the part of the realized breach left uncovered by the equity, $(B_t - E_{t-})^+$, is covered by the surviving members through the default fund, which they have to refill by the following rule, at each $t = \tau_Z^{\delta}$ with $\tau_Z < \bar{T}$:
$\epsilon_t^i = (B_t - E_{t-})^+ \frac{J_t^i DF_t^i}{\sum_{j \in N} J_t^j DF_t^j}$ proportional to their default fund margins
  - or other keys of repartition such as initial margins, sizes of the positions, expected shortfall allocation (see Armenti, Crépey, Drapeau, and Papapantoleon (2015)),...



Outline

- We refer to the member 0 as "the member" henceforth, the other members being collectively referred to as "the clearinghouse"
- For notational simplicity, we remove any index 0 referring to the member.
- For the member, the effective time horizon of the problem is $\bar{\tau}^\delta = \mathbb{1}_{\tau<\bar{T}}\tau^\delta + \mathbb{1}_{\{\tau\geq\bar{T}\}}\bar{T}$
- We assume that
  - variation margins are remunerated at a flat OENIA rate r_t
  - initial margins and default fund contributions are remunerated at the rate $(r_t + c_t)$ with $c_t < 0$, e.g. $c_t = -20$ bp
  - the member can invest (respectively get unsecured funding) at a rate $(r_t + \lambda_t)$ (respectively $(r_t + \bar{\lambda}_t)$)

Following Green, Kenyon, and Dennis (2014), we model the cost of the regulatory capital required for being part of the clearinghouse as $k_t K_t dt$

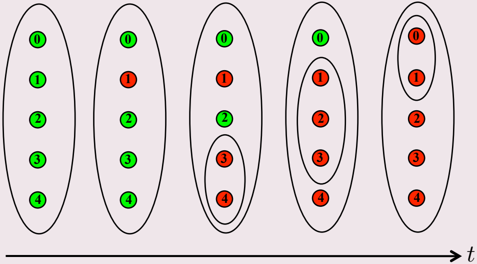
- K_t is the CCP regulatory capital of the member,
- k_t is a proportional hurdle rate

Marshall-Olkin Model of Default Times

- We model credit dependence between members through joint defaults
    - "Instantaneous contagion"
- Marshall-Olkin copula model of the default times $\tau_i, i \in N$
- Define a family $\mathcal{Y}$ of shocks, i.e. subsets $Y \subseteq N$ of obligors, usually consisting of the singletons $\{0\}$, $\{1\}$, $\dots$, $\{n\}$ and a few common shocks representing simultaneous defaults
- Define, for $Y \in \mathcal{Y}$, independent γ_Y exponential random variables ϵ_Y
- Set, for each i,

$$\tau_i = \bigwedge_{Y \in \mathcal{Y};\, Y \ni i} \eta_Y$$

Example: $n = 5$ and $\mathcal{Y} = \{\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{3, 4\}, \{1, 2, 3\}, \{0, 1\}\}$.



→ Pre-default intensity of the member: $\gamma_\bullet = \sum_{Y \in \mathcal{Y}_\bullet} \gamma_Y$, where $\mathcal{Y}_\bullet = \{Y \in \mathcal{Y}; 0 \in Y\}$.

CCVA Formula

Theorem

First order, linearized CCVA at time 0:

$$\widehat{\Theta}_0 = \mathbb{E}\Big[\sum_{0<\tau_Z^\delta<\bar{\tau}} \beta_{\tau_Z^\delta}\epsilon_{\tau_Z^\delta} + \int_0^{\bar{\tau}} \beta_s \widehat{f}_s(0)ds\Big] = \underbrace{\mathbb{E}\sum_{0<\tau_Z^\delta<\bar{\tau}} \beta_{\tau_Z^\delta}\epsilon_{\tau_Z^\delta}}_{CVA} + \underbrace{\mathbb{E}\int_0^{\bar{\tau}} \beta_s dva_s ds}_{DVA}$$

$$+ \underbrace{\mathbb{E}\int_0^{\bar{\tau}} \beta_s\Big(-c_s(C_s - P_{\widehat{s}-}) + \widetilde{\lambda}_s(P_s - C_s)^- - \lambda_s(P_s - C_s)^+\Big)ds}_{FVA} + \underbrace{\mathbb{E}\int_0^{\bar{\tau}} \beta_s k_s K_s ds}_{KVA},$$

where

- $dva = -\gamma\widehat{\xi}$, where $\widehat{\xi}$ is a predictable process such that $\widehat{\xi}_\tau = \mathbb{E}(\beta_\tau^{-1}\beta_{\tau^\delta}\xi \,|\, \mathcal{G}_{\tau-})$, with $\xi = (1-R)(Q_{\tau^\delta} - C_\tau)^+$, so that the DVA can be ignored by setting $R = 1$.
- $\widetilde{\lambda} = \bar{\lambda} - (1-\bar{R})\gamma_\bullet$, in which the DVA2 can be ignored by setting $\bar{R} = 1$.

For numerical purposes, we use the following randomized version of the theorem:

Corollary

Given an independent μ-exponential time ζ,

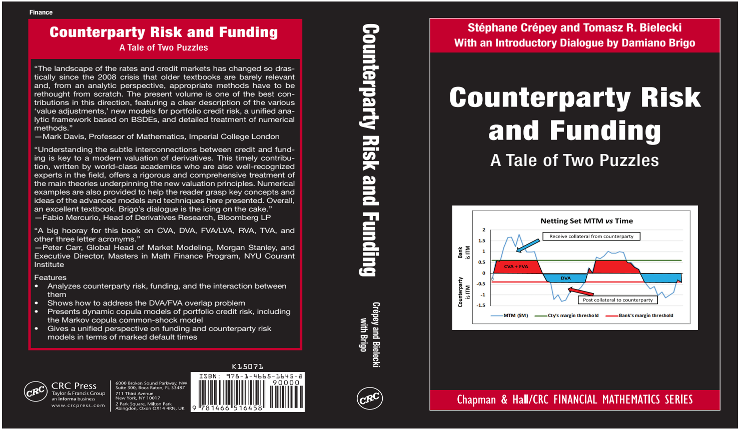
$$\widehat{\Theta}_0 = \mathbb{E}\Big\{ \sum_{0<\tau_Z^\delta<\bar{\tau}} \beta_{\tau_Z^\delta} \epsilon_{\tau_Z^\delta} + \mathbb{1}_{\{\zeta<\bar{\tau}\}} \frac{e^{\mu\zeta}}{\mu} \beta_\zeta \widehat{f}_\zeta(0) \Big\}$$

$$= \mathbb{E}\Big\{ \sum_{0<\tau_Z^\delta<\bar{\tau}} \beta_{\tau_Z^\delta} \epsilon_{\tau_Z^\delta} + \mathbb{1}_{\{\zeta<\bar{\tau}\}} \frac{e^{\mu\zeta}}{\mu} \times \Big[-\beta_{\zeta^\delta} \gamma_\bullet (1-R)\big(Q_{\zeta^\delta} - C_\zeta\big)^+$$

$$+ \beta_\zeta \Big(-c_\zeta (C_\zeta - P_{\widehat{\zeta}_-}) + \widetilde{\lambda}_\zeta (P_\zeta - C_\zeta)^- - \lambda_\zeta (P_\zeta - C_\zeta)^+ + k_\zeta K_\zeta \Big) \Big] \Big\}.$$

Outline

- Bilateral trading (CSA) setup between a bank, say the member, labeled 0, in the above CCVA setup, and a counterparty, say another member $i \neq 0$



- Let VM denote the variation margin, where $VM \geq 0$ (resp. ≤ 0) means collateral posted by the bank and received by the counterparty (resp. posted by the counterparty and received by the bank)
- Let $IM^b \geq 0$ (resp. $IM^c \leq 0$) represent the initial margin posted by the bank (resp. the negative of the initial margin posted by the counterparty)

→ $C^b = VM + IM^b$ and $C^c = VM + IM^c$ represent respectively the collateral guarantee for the counterparty and the negative of the collateral guarantee for the bank

- Assuming all the margins re-hypothecable in the bilateral setup, the collateral funded by the bank is $C = VM + IM^b + IM^c$

BVA Formula

Theorem (Crépey and Song (2015))

First order, linearized BVA at time 0:

$$\bar{\Theta}_0 = \mathbb{E}\Big[\int_0^{\bar\tau} \beta_s \bar{f}_s(0)ds\Big] = \underbrace{\mathbb{E}\int_0^{\bar\tau} \beta_s cdva_s ds}_{CDVA} +$$

$$+ \underbrace{\mathbb{E}\int_0^{\bar\tau} \beta_s \Big(- c_s(C_s - P_{\widehat{s}-}) + \widetilde{\lambda}_s\,(P_s - C_s)^- - \lambda_s\,(P_s - C_s)^+ \Big) ds}_{FVA} + \underbrace{\mathbb{E}\int_0^{\bar\tau} \beta_s k_s K_s ds}_{KVA}$$

- P means the mark-to-market of the position of the member with the counterparty i (viewed from the perspective of the latter),
- the meaning of β, c, λ, $\widetilde{\lambda}$, k and K is as in the CCVA setup, but "$c = 0$" and the formula for the regulatory capital K is different,
- $\tau = \tau_b \wedge \tau_c$ is the first-to-default time of the bank and its counterparty (as opposed to the default time of the member previously)

- $cdva = \gamma\widehat{\xi}$, where $\widehat{\xi}$ is a predictable process such that $\widehat{\xi}_\tau = \mathbb{E}(\beta_\tau^{-1}\beta_{\tau^\delta}\xi \,|\, \mathcal{G}_{\tau-})$, with

$$\xi = \mathbb{1}_{\{\tau_c \le \tau_b^\delta\}}(1-R_c)(Q_{\tau^\delta} - C_\tau^c)^- - \mathbb{1}_{\{\tau_b \le \tau_c^\delta\}}(1-R_b)(Q_{\tau^\delta} - C_\tau^b)^+,$$

  in which the recovery rates R_c of the counterparty to the bank and R_b of the bank to the counterparty are usually taken as 40%.

For numerical purposes, we use the following randomized version of this theorem, with $\mathcal{Y}_b = \{Y \in \mathcal{Y};\, 0 \in Y\}$, $\mathcal{Y}_c = \{Y \in \mathcal{Y};\, i \in Y\}$.

Corollary

Given an independent μ-exponential time ζ,

$$\bar{\Theta}_0 = \mathbb{E}\Big\{\mathbb{1}_{\{\zeta<\bar{\tau}\}}\frac{e^{\mu\zeta}}{\mu}\beta_\zeta \bar{f}_\zeta(0)\Big\}$$

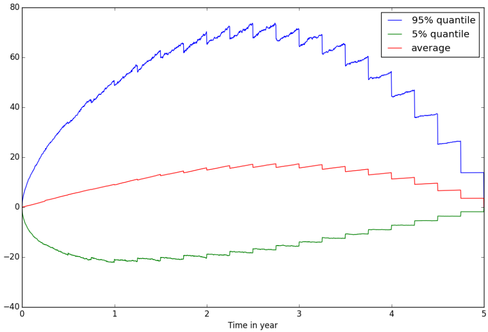
$$= \mathbb{E}\Big\{\mathbb{1}_{\{\zeta<\bar{\tau}\}}\frac{e^{\mu\zeta}}{\mu}\Big[\beta_{\zeta^\delta}\Big(\Big(\sum_{Y\in\mathcal{Y}_c}\gamma_Y + \mathbb{1}_{\{\tau_c\le\zeta^\delta\}}\sum_{Y\in\mathcal{Y}_b\setminus\mathcal{Y}_c}\gamma_Y\Big)(1-R_c)(Q_{\zeta^\delta} - C_\zeta^c)^-$$

$$-\Big(\sum_{Y\in\mathcal{Y}_b}\gamma_Y + \mathbb{1}_{\{\tau_b\le\zeta^\delta\}}\sum_{Y\in\mathcal{Y}_c\setminus\mathcal{Y}_b}\gamma_Y\Big)(1-R_b)(Q_{\zeta^\delta} - C_\zeta^b)^+\Big)$$

$$+\beta_\zeta\Big(-c_\zeta(C_\zeta - P_{\widehat{\zeta}-}) + \widetilde{\lambda}_\zeta(P_\zeta - C_\zeta)^- - \lambda_\zeta(P_\zeta - C_\zeta)^+ + k_\zeta K_\zeta\Big)\Big]\Big\}.$$

Outline

- Black-Scholes stock S with historical drift μ and volatility σ,
- Asset swap with cash-flows $\frac{1}{4}(S_{T_{l-1}} - K)$ at increasing quarters T_l, $l = 1, \ldots, d$



- Notional for this swap such that the time-0 value of each leg of the swap is €1 (y axis in % above)

- We consider a subset of nine representative members of the CDX index, with CDS spreads (average 3 year and 5 year bp spread) shown in increasing order in the first row of the following table.

(*Top*) Average 3 and 5 year CDS spreads for a representative subset of nine members of the CDX index as of 17 December 2007.
(*Bottom*) Coefficients α_i summing up to 0 used for determining the positions in the swap of the nine members.

Σ	45	52	56	61	73	108	176	367	1053
α	(0.46)	0.09	0.23	(0.05)	0.34	(0.04)	0.69	(0.44)	(0.36)

- The role of the reference member 0 will be played alternately by each of the nine members in the above table, for positions in the swap determined by the coefficients α_i summing up to zero through the rule $\omega_i = -\frac{\alpha_i}{\alpha_0}$

- We compare two trading setups:
  - A bilateral CSA setup where the member 0 trades a long $\omega_i \in \mathbb{R}$ swap units position separately with each member $i \neq 0$
  - A CCP setup where each member $i \in N$ trades a short $\omega_i \in \mathbb{R}$ swap units position through the CCP
- In each considered case, the reference member 0 has an aggregated long one unit net position in the swap, and a gross position (compression factor)

$$\nu_0 = \sum_{i \neq 0} |\omega_i^{csa}| = \sum_{i \neq 0} \frac{|\alpha_i|}{|\alpha_0|} = \frac{\sum_{i \in N} |\alpha_i|}{|\alpha_0|} - 1,$$

so the smaller $|\alpha_0|$, the bigger the compression factor ν_0.

- In the CCP setup, IM^i (resp. $IM^i + DF^i$) set as the value at risk of level a_{im} (resp. a_m) of the variation-margined $P\&L^i$
- In the CSA setup, initial margin IM^i set as the value at risk of level $a'_{im} = a_m$ of the variation-margined $P\&L^i$
- In both setups a value at risk of level $a_{ead} > a'_{im} = a_m$ is used for computing the exposure at defaults in the regulatory capital formulas

Netting Benefit

ν_0	2.91	4.87	5.14	6.50	6.94	10.74	29.00	53.00	66.50
α_0	0.69	(0.46)	(0.44)	(0.36)	0.34	0.23	0.09	(0.05)	(0.04)
Σ_0	176	45	367	1053	73	56	52	61	108
CVA	9.41	15.92	12.15	8.66	22.64	34.01	88.36	150.49	187.44
DVA	(5.48)	(2.45)	(20.76)	(63.73)	(5.53)	(7.19)	(17.67)	(36.02)	(79.52)
FVA	9.01	3.99	27.45	74.77	9.61	10.27	27.40	64.35	140.13
KVA^{ccr}	10.02	18.92	17.59	18.79	25.87	40.93	108.76	197.83	246.67
KVA^{cva}	4.24	8.12	7.50	8.30	11.44	18.25	47.79	85.72	107.13
BVA	32.69	46.95	64.69	110.53	69.56	103.47	272.30	498.40	681.36
CVA	5.18	8.67	5.02	2.48	7.38	8.29	8.24	8.84	7.12
DVA	(2.05)	(0.55)	(4.18)	(10.06)	(0.88)	(0.70)	(0.64)	(0.74)	(1.29)
FVA	10.66	3.68	20.83	46.74	5.16	4.40	4.12	4.50	7.04
KVA	0.19	0.19	0.18	0.14	0.19	0.19	0.19	0.19	0.19
CCVA	16.03	12.54	26.02	49.37	12.73	12.88	12.55	13.54	14.34

Impact of the Credit Spread of the Reference Member

ν_0	4.87	29.00	10.74	53.00	6.94	66.50	2.91	5.14	6.50
α_0	(0.46)	0.09	0.23	(0.05)	0.34	(0.04)	0.69	(0.44)	(0.36)
Σ_0	45	52	56	61	73	108	176	367	1053
CVA $/\nu_0$	3.27	3.05	3.17	2.84	3.26	2.82	3.23	2.37	1.33
DVA$/\nu_0$	(0.50)	(0.61)	(0.67)	(0.68)	(0.80)	(1.20)	(1.88)	(4.04)	(9.80)
FVA $/\nu_0$	0.82	0.94	0.96	1.21	1.38	2.11	3.09	5.34	11.50
KVA$/\nu_0$	5.55	5.40	5.51	5.35	5.38	5.32	4.90	4.89	4.17
BVA$/\nu_0$	9.64	9.39	9.63	9.40	10.02	10.25	11.22	12.59	17.01
CVA	8.67	8.24	8.29	8.84	7.38	7.12	5.18	5.02	2.48
DVA	(0.55)	(0.64)	(0.70)	(0.74)	(0.88)	(1.29)	(2.05)	(4.18)	(10.06)
FVA	3.68	4.12	4.40	4.50	5.16	7.04	10.66	20.83	46.74
KVA	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.18	0.14
CCVA	12.54	12.55	12.88	13.54	12.73	14.34	16.03	26.02	49.37

Impact of the liquidation period

Member	61 bps, $\nu_0 = 53.00$		367 bps, $\nu_0 = 5.14$	
δ	5d	15d	5d	15d
CVA $/\nu_0$	1.58	2.84	1.31	2.36
DVA $/\nu_0$	(0.38)	(0.68)	(2.25)	(4.04)
FVA $/\nu_0$	0.41	1.21	1.73	5.34
KVA $/\nu_0$	3.19	5.35	2.90	4.88
BVA $/\nu_0$	5.18	9.40	5.94	12.59
CVA	8.84	13.62	5.02	7.60
DVA	(0.74)	(1.28)	(4.18)	(7.58)
FVA	4.50	7.85	20.83	36.35
KVA	0.19	0.32	0.18	0.30
CCVA	13.54	21.80	26.02	44.25

Impact of the level of the quantiles that are used for setting initial margins, default fund contributions and exposures at default (with $a_m = a'_{im}$ everywhere)

Member	$\Sigma_0 = 61$bp, $\nu_0 = 53.00$			$\Sigma_0 = 367$bp, $\nu_0 = 5.14$		
	$a_{ead} = 85\%$	$a_{ead} = 95\%$	$a_{ead} = 99,7\%$	$a_{ead} = 85\%$	$a_{ead} = 95\%$	$a_{ead} = 99,7\%$
	$a'_{im} = 80\%$	$a'_{im} = 90\%$	$a'_{im} = 99\%$	$a'_{im} = 80\%$	$a'_{im} = 90\%$	$a'_{im} = 99\%$
CVA / ν_0	2,84	1,25	0,14	2,36	1,04	0,12
DVA / ν_0	(0,68)	(0,30)	(0,03)	(4,04)	(1,79)	(0,21)
FVA / ν_0	1,21	1,34	1,80	5,34	6,01	8,24
KVA^{ccr} / ν_0	3,73	7,00	8,20	3,42	6,45	7,62
KVA^{cva} / ν_0	1,62	3,03	3,54	1,46	2,75	3,24
BVA / ν_0	9,40	12,62	13,67	12,59	16,24	19,22
	$a_{im} = 70\%$	$a_{im} = 80\%$	$a_{im} = 95\%$	$a_{im} = 70\%$	$a_{im} = 80\%$	$a_{im} = 95\%$
CVA	8,84	5,52	1,69	5,02	3,11	0,93
DVA	(0,74)	(0,32)	(0,03)	(4,18)	(1,83)	(0,19)
FVA	4,50	6,74	12,15	20,83	31,21	56,34
KVA	0,19	0,36	0,43	0,18	0,33	0,39
CCVA	13,54	12,62	14,27	26,02	34,66	57,66

When higher quantile levels are used for the margins and exposures at default, we observe :

- The same qualitative patterns as before in terms of the comparison between the CSA and the CCP setup, which is mainly driven by the compression factor ν_0.
- Inside each setup (CSA or CCP), an expected
  shift from CVA and DVA into KVA (resp. FVA) in the CSA (resp. CCP) se
  - Ultimately, for very high quantiles, CVA and DVA would reach zero whereas KVA and FVA would keep increasing, meaning that excessive margins become useless and a pure cost to the system, in the CSA as in the CCP setup

Conclusions

- We developed a rigorous theoretical comparison between bilateral and centrally cleared trading
- This theoretical framework can be used by a clearinghouse to
  - Analyze the benefit for a dealer to trade centrally as a member, rather than on a bilateral basis
  - Find the right balance between initial margins and default fund in order to minimize this cost, hence become more competitive
  - Help its members risk manage their CCVA
- We illustrate the netting benefit of CCPs
- Transfer of CVA and/or KVA into FVA when switching from a bilateral CSA to a CCP setup
- Potentially important uncovered issues:
  - Fragmentation in case of several CCPs and/or markets
  - Defaultability of the CCP
  - Cost of more realistic liquidation procedures
  - Market incompleteness
  - Wrong way risk, ...

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