Dynamic optimal execution in a mixed-market-impact Hawkes price model

Aurélien Alfonsi

Ecole des Ponts - Paris-Est University
Joint work with Pierre Blanc

September 26, 2015

London-Paris Bachelier Workshop on Mathematical Finance 2015

September 26, 2015

Price impact modeling : a matter of time scale

The price impact : how market orders modify the price of the traded assets.

- At a very low frequency, the price impact is usually ignored.
- At a very high frequency, the price impact is built in the Limit Order Book dynamics.
- At a mesoscopic time scale, one has to model it : trade-off between tractability and realism that takes into account the trade frequency. One typically wants to solve the optimal execution problem : how to buy/sell optimally a given amount of assets within the deadline $T > 0$?

Obizhaeva and Wang model (2005,2013)

One large trader that trades on $[0, T]$. Other market orders create noise $\to S_t^0$ martingale.

- Let $q > 0$ and $\epsilon \in [0, 1]$. Asset price : $P_t = S_t^0 + \frac{\epsilon}{q}(X_t - X_0) + D_t$, with

$$dD_t = -\rho D_t dt + \frac{1-\epsilon}{q} dX_t,$$

  and $D_0 = 0$ (steady state).
- Cost of trading dX_t :

$$dX_t \times (P_t + \frac{1}{2q} dX_t).$$

- Optimal liquidation strategy ($X_T = 0$) that minimizes the expected cost :

$$dX_t = -\frac{X_0}{2 + \rho T}[\delta_0(dt) + \rho dt + \delta_T(dt)].$$

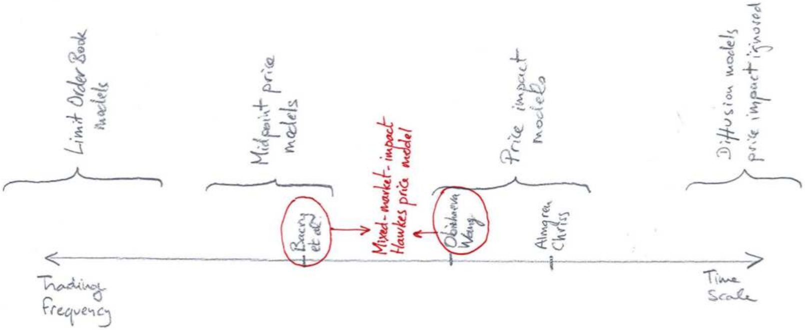
Comments on the OW model

- Possible extensions to nonlinear price impact or non exponential decay Kernel.
- Almgren and Chriss model is a limit of OW model when $\rho \to \infty$ or equivalently when the trading frequency decreases to zero.
- The price resilience ρ can be seen as the feedback of market makers.
- Impact of other market orders modeled through S_t^0. No resilience for them.

High frequency price models

- Limit Order Book models : Abergel and Jedidi, Cont and De Larrard, Huang, Lehalle and Rosenbaum,... the LOB dynamics encompasses price impact.
- There are many more LOB events than price changes
  $\rightarrow$ Midpoint price models : Bacry, Delattre, Hoffmann and Muzy, Bacry and Muzy, Robert and Rosenbaum,... These models are usually meant to reproduce statistical properties of the price. The price impact is generally not directly modelled.

Position of our work



The price model : OW with a flow of orders

N_t : sum of the signed volumes ($dN_t > 0$ if buy order) of past market orders on the book between time 0 and time t.
Assumption : N is a càdlàg process, $(\mathcal{F}_t)$-adapted
$\forall t > 0,\ \sup_{s\in[0,t]} \mathbb{E}[N_s^2] < \infty$.

$$P_t = \underbrace{S_t}_{\text{fundamental price}} + \underbrace{D_t}_{\text{mesoscopic price deviation}},$$

$$\begin{aligned} \mathrm{d}S_t &= \frac{\nu}{q} \underbrace{\mathrm{d}N_t}_{\text{market orders}} \\ \mathrm{d}D_t &= \underbrace{-\rho\, D_t\, \mathrm{d}t}_{\text{market resilience}} + \frac{1-\nu}{q} \underbrace{\mathrm{d}N_t}_{\text{market orders}}. \end{aligned}$$

Rk : Adding a martingale S_t^0 to P_t does not change what follows.

The liquidation strategy

A "strategic" trader with X_t assets at time t. We assume that

- X is a càglàd, i.e. he can react instantly to other market orders,
- X is $(\mathcal{F}_t)$-adapted, sq. integrable with bounded variation,
- $X_0 \in \mathbb{R}$, $X_{T+} = 0$.

Price : $P_t = S_t + D_t$ with $\epsilon \in [0,1]$,

$$dS_t = \frac{1}{q}\left(\nu \mathrm{d}N_t + \epsilon \mathrm{d}X_t\right), \; dD_t = -\rho\, D_t\, \mathrm{d}t + \frac{1}{q}\left((1-\nu)\mathrm{d}N_t + (1-\epsilon)\mathrm{d}X_t\right).$$

The processes P, S, D are làdlàg :

$$S_t - S_{t-} = \frac{\nu}{q}(N_t - N_{t-}),\; S_{t+} - S_t = \frac{\epsilon}{q}(X_{t+} - X_t),$$

$$D_t - D_{t-} = \frac{1-\nu}{q}(N_t - N_{t-}),\; D_{t+} - D_t = \frac{1-\epsilon}{q}(X_{t+} - X_t).$$

The liquidation cost

- Cost of the trade dX_t : $dX_t \times (P_t + \frac{1}{2q} dX_t) = dX_t \times \frac{P_t + P_{t+}}{2}$.
- Cost of the strategy :

$$C(X) = \int_{[0,T)} P_u \, \mathrm{d}X_u \; + \; \frac{1}{2q} \sum_{0 \le \tau < T} (\Delta X_\tau)^2 \; - \; P_T X_T \; + \; \frac{1}{2q} X_T^2$$

Optimal execution problem : find the liquidation strategy X that minimizes $\mathbb{E}[C(X)]$.

Price Manipulation Strategies

Definition 1

A Price Manipulation Strategy (PMS) in the sense of Huberman and Stanzl is a X such that $X_0 = X_{T+} = 0$ a.s. and $\mathbb{E}[C(X)] < 0$.

Theorem 2

The model does not admit PMS if, and only if the process P is a $(\mathcal{F}_t)$-martingale when $X \equiv 0$. Then, the optimal strategy X^* is

$$\Delta X_0^* = -\frac{x_0}{2+\rho T},\ \Delta X_T^* = -\frac{x_0}{2+\rho T},\ \mathrm{d}X_t^* = -\rho\frac{x_0}{2+\rho T}\mathrm{d}t \text{ for } t \in (0,T),$$

and has the expected cost $\mathbb{E}[C(X^*)] = -P_0 x_0 + \left[\frac{1-\epsilon}{2+\rho(T-t)} + \frac{\epsilon}{2}\right] x_0^2/q.$

This is the same strategy as in the OW model.
Question : which flows of orders satisfy this condition ?

Modeling of the order flow

The Mixed-market-Impact Hawkes (MIH) model. $N_t = N_t^+ - N_t^-$.
Jump law μ, respective intensities κ_t^+ and κ_t^- : càdlàg processes that follow the Markovian marked Hawkes dynamics

$$\begin{aligned} d\kappa_t^+ &= -\beta\,(\kappa_t^+ - \kappa_\infty)\,dt + \varphi_s(dN_t^+/m_1) + \varphi_c(dN_t^-/m_1), \\ d\kappa_t^- &= -\beta\,(\kappa_t^- - \kappa_\infty)\,dt + \varphi_c(dN_t^+/m_1) + \varphi_s(dN_t^-/m_1), \end{aligned}$$

where $\varphi_s, \varphi_c : \mathbb{R}^+ \to \mathbb{R}^+$ are measurable positive functions that satisfy $\iota_s = \int_{\mathbb{R}+} \varphi_s(v/m_1)\mu(dv) < \infty$, $\iota_c = \int_{\mathbb{R}+} \varphi_c(v/m_1)\mu(dv) < \infty$, $\int_{\mathbb{R}+} \varphi_s^2(v/m_1)\mu(dv) < \infty$ and $\int_{\mathbb{R}+} \varphi_c^2(v/m_1)\mu(dv) < \infty$.
We define $\alpha = \iota_s - \iota_c$, $\delta_t = \kappa_t^+ - \kappa_t^-$, $\Sigma_t = \kappa_t^+ + \kappa_t^-$ and
$I_t = \int_0^t [(\varphi_s - \varphi_c)(dN_u^+/m_1) - (\varphi_s - \varphi_c)(dN_u^-/m_1)]$.

- Stationary iff $\iota_s + \iota_c < \beta$.
- $\iota_c = \iota_s = \beta = 0 \ \to$ Poisson model.
- $\nu = 1$, $\varphi_s(x) = \iota_s$, $\iota_c = 0$ and $\mu(dx) = \delta_1(dx) \ \to$ price model proposed in Bacry, Delattre, Hoffmann and Muzy.

The optimal liquidation strategy I

Let $\epsilon \in [0,1)$. The optimal strategy to liquidate x_0 is explicit. It is a linear combination of $(x_0, D_0, \delta_0, I, N)$ and can be written as

$$X^* = X^{\mathrm{OW}} + X^{\mathrm{trend}} + X^{\mathrm{dyn}},$$

where

$$\Delta X_0^{\mathrm{OW}} = -\frac{x_0}{2+\rho T},\ \Delta X_T^{\mathrm{OW}} = -\frac{x_0}{2+\rho T},\ \mathrm{d}X_t^{\mathrm{OW}} = -\rho\frac{x_0}{2+\rho T}\mathrm{d}t \text{ for } t \in (0,T),$$

is the part that is proportional to x_0 and is the Obizhaeva and Wang strategy,

The optimal liquidation strategy II

$$
\begin{aligned}
(1-\epsilon)\Delta X_0^{\text{trend}} &= \frac{\frac{\delta_0 m_1}{2\rho}\times[2+\rho T\times\{1+\zeta(\eta T)+\nu\rho[1-\zeta(\eta T)]/\eta\}]-[1+\rho T]qD_0}{2+\rho T},\\
(1-\epsilon)\Delta X_T^{\text{trend}} &= \frac{\delta_0 m_1}{2\rho}\times\left[\frac{2+\rho T\times\{1+\zeta(\eta T)+\nu\rho[1-\zeta(\eta T)]/\eta\}}{2+\rho T}-2\rho\,\Phi_\eta(0,T)-2\exp(-\beta T)\right]\\
&\quad+\ \frac{qD_0}{2+\rho T},
\end{aligned}
$$

and, on $(0,T)$,

$$
\begin{aligned}
(1-\epsilon)\mathrm{d}X_t^{\text{trend}} =& \frac{\delta_0 m_1}{2\rho}\times\left[\frac{2+\rho T\times\{1+\zeta(\eta T)+\nu\rho[1-\zeta(\eta T)]/\eta\}}{2+\rho T}-2\rho\,\Phi_\eta(0,t)\right.\\
&\left.-2\phi_\eta(t)\exp(-\beta t)\right]\rho\mathrm{d}t+\frac{qD_0}{2+\rho T}\,\rho\mathrm{d}t.
\end{aligned}
$$

This is the part that is proportional to (D_0,δ_0) and takes advantage of the initial trend.

The optimal liquidation strategy III

$$
\begin{aligned}
(1-\epsilon)\Delta X_0^{\mathrm{dyn}} =& 0, \\
(1-\epsilon)\Delta X_T^{\mathrm{dyn}} =& -\ m_1 \left[\Theta_{\chi_T}\ \Phi_\eta\left(\tau_{\chi_T}, T\right)\ +\ \sum_{i=1}^{\chi_T - 1} \Theta_i\ \Phi_\eta(\tau_i, \tau_{i+1}) \right]\ +\ \sum_{0<\tau\leq T} \frac{(1-\nu)\ \Delta N_\tau}{2+\rho(T-\tau)} \\
& + \frac{m_1}{2\rho} \times \sum_{0<\tau\leq T} \frac{2+\rho(T-\tau)\times\{1+\zeta(\eta(T-\tau))+\nu\rho[1-\zeta(\eta(T-\tau))]/\eta\}}{2+\rho(T-\tau)} \Delta I_\tau \\
& -\ \frac{m_1}{\rho}\ \Theta_{\chi_T} \exp(-\beta T),
\end{aligned}
$$

The optimal liquidation strategy IV

and, on $(0,T)$,

$$
\begin{aligned}
(1-\epsilon)\mathrm{d}X_t^{\mathrm{dyn}} = & -m_1\,\phi_\eta(t)\,\Theta_{\chi_t}\exp(-\beta t)\,\mathrm{d}t + \left[\sum_{0<\tau\le t}\frac{(1-\nu)\Delta N_\tau}{2+\rho(T-\tau)}\right]\rho \mathrm{d}t \\
& + \left[\sum_{0<\tau\le t}\frac{2+\rho(T-\tau)\times\{1+\zeta(\eta(T-\tau))+\nu\rho[1-\zeta(\eta(T-\tau))]/\eta\}}{2+\rho(T-\tau)}\,\Delta I_\tau\right]\frac{m_1}{2}\,\mathrm{d}t \\
& - \left[\Theta_{\chi_t}\,\Phi_\eta\left(\tau_{\chi_t},t\right) + \sum_{i=1}^{\chi_t-1}\Theta_i\,\Phi_\eta(\tau_i,\tau_{i+1})\right]\rho m_1\,\mathrm{d}t
\end{aligned}
$$

$$
+\ \frac{1+\rho(T-t)}{2+\rho(T-t)}\left\{\frac{m_1}{\rho}\,\mathrm{d}I_t-(1-\nu)\,\mathrm{d}N_t\right\} + \frac{m_1}{2\rho}(\nu\rho-\eta)\times\frac{\rho(T-t)\times[1-\zeta(\eta(T-t))]/\eta}{2+\rho(T-t)}\,\mathrm{d}I_t.
$$

This is the part that is proportional to the processes N and I and gives the optimal reaction to other trades.

The optimal liquidation strategy V

The value function of the problem is then :

$$q \times \mathcal{C}(t,x,d,z,\delta,\Sigma) = -q(z+d)x + \left[\frac{1-\epsilon}{2+\rho(T-t)} + \frac{\epsilon}{2}\right]x^2 + \frac{\rho(T-t)}{2+\rho(T-t)}\left[qd - \mathcal{G}_\eta(T-t)\frac{\delta m_1}{\rho}\right]x$$

$$-\frac{1}{1-\epsilon} \times \frac{\rho(T-t)/2}{2+\rho(T-t)}\left[qd - \mathcal{G}_\eta(T-t)\frac{\delta m_1}{\rho}\right]^2 + \hat{c}_\eta(T-t)\left(\frac{\delta m_1}{\rho}\right)^2$$

$$+ e(T-t)\Sigma + g(T-t),$$

where for $u \in [0,T]$, $\mathcal{G}_\eta(u) = \zeta(\eta u) + \nu\rho[1-\zeta(\eta u)]/\eta$,
$\hat{c}_\eta(u) = -\frac{1}{1-\epsilon} \times \left(1 - \frac{\nu\rho}{\eta}\right)^2 \times \frac{\rho u \zeta(\eta u)}{8} \times [1 + \exp(-\eta u) - 2\zeta(\eta u)].$

The optimal liquidation strategy VI

$$
\begin{aligned}
\zeta(y) &= \frac{1-\exp(-y)}{y}, \omega(y) = \frac{1-\zeta(y)}{y} \\
L(r,\lambda,t) &= r\int_0^t \frac{\exp(\lambda s)}{2+rs}\,\mathrm{d}s \;=\; \exp(-2\lambda/r)\;\left[\mathcal{E}\left(\frac{\lambda}{r}(2+rt)\right) - \mathcal{E}\left(\frac{2\lambda}{r}\right)\right] \\
\phi_\eta(t) &= \frac{1}{2(2+\rho(T-t))} \times \Big[1+\exp(-\eta(T-t)) + \nu\rho(T-t)\zeta(\eta(T-t)) \\
&\quad + \frac{\beta}{\rho}\left[2+\rho(T-t)\times\{1+\zeta(\eta(T-t))+\nu\rho[1-\zeta(\eta(T-t))]/\eta\}\right]\Big],\ \eta \neq 0, \\
\Phi_\eta(s,t) &= \frac{1}{2}\left(\frac{1}{\rho}+\frac{\nu}{\eta}\right)\times[\exp(-\beta s)-\exp(-\beta t)] \\
&\quad + \frac{\exp(-\beta T)}{2\rho}\times\left[1+\frac{\nu(\rho-2\beta)}{\eta}+\frac{\beta}{\eta}\left(1-\frac{\nu\rho}{\eta}\right)\right]\times[L(\rho,\beta,T-s)-L(\rho,\beta,T-t)] \\
&\quad + \frac{\exp(-\beta T)}{2\rho}\times\left[1-\frac{\nu\rho}{\eta}-\frac{\beta}{\eta}\left(1-\frac{\nu\rho}{\eta}\right)\right]\times[L(\rho,\alpha,T-s)-L(\rho,\alpha,T-t)],\ \eta\neq 0.
\end{aligned}
$$

Comments

- Only depends on (φ_s, φ_c) through $\varphi_s - \varphi_c$.
- Due to the self-exciting behaviour, the reaction to other market orders is not always in the opposite direction and depend on the order size.
- The optimal strategy X^* satisfies a.s., dt-a.e. on $(0, T)$,

$$(1-\epsilon)X_t^* = -[1+\rho(T-t)]D_t^* \quad (1)$$
$$+\frac{m_1}{2\rho} \times [2+\rho(T-t) \times \{1+\zeta(\eta(T-t)) + \nu\rho(T-t)\omega(\eta(T-t))\}]\, \delta_t.$$

- The quantity to liquidate that maximizes $\mathbb{E}[C(X)] + P_0 \times x_0$, i.e. the expected liquidation cost with respect to the mark-to-market value is

$$x_0^* = \frac{\rho T[qD_0 - \mathcal{G}_\eta(T)\frac{\delta_0 m_1}{\rho}]}{2\left(1+\frac{\epsilon}{2}\rho T\right)}.$$

Numerical example

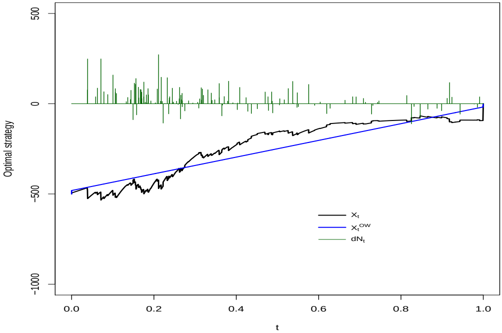


FIGURE: Optimal strategy $(X_t^*)_{0\le t\le T}$, for $q = 100,\ T = 1,\ \beta = 20,\ \iota_s = 16,$ $\iota_c = 2,\ \kappa_\infty = 12, \epsilon = 0.3,\ \nu = 0.3,\ D_0 = 0.1,\ \kappa_0^+ = \kappa_0^- = 60,\ m_1 = 50,\ X_0 =$ $-500,\ \mu = \mathrm{Exp}(1/m_1),\ \varphi_s(y) = 1.2\times y^{0.2} + 0.5\times y^{0.7} + 14.4\times y,\ \varphi_c(y) =$ $1.2\times y^{0.2} + 0.5\times y^{0.7} + 0.4\times y$ for all $y > 0$.

The Mixed-Impact Hawkes Martingale (MIHM) model

Let $\mathcal{S}(\mu) = \{y \geq 0, m_1 \times y \text{ is in the support of } \mu\}$.

Proposition 3

The MIH model does not admit PMS if, and only if

$$\beta = \rho,\ \alpha = (1-\nu)\rho,\ \varphi_s(x) - \varphi_c(x) = \alpha x \text{ for } x \in \mathcal{S}(\mu),\ \text{and } qD_0 = \frac{m_1}{\rho}\delta_0$$

or $\mu = Dirac(0)$ *with* $D_0 = 0$*. The optimal execution strategy is then the same as in the OW model.*

Proof $\delta_t = \kappa_t^+ - \kappa_t^-$ satisfies $\mathrm{d}\delta_t = -\beta\, \delta_t\, \mathrm{d}t + \mathrm{d}I_t$.
$\mathrm{d}P_t = -\rho D_t dt + \frac{1}{q}\mathrm{d}N_t = \frac{1}{q}(\mathrm{d}N_t - \delta_t m_1 \mathrm{d}t) + \left(\frac{m_1}{q}\delta_t - \rho D_t\right)\mathrm{d}t.$
Thus, P is a mg $\iff$ $\frac{m_1}{\rho}\delta_t = qD_t$, and we use $dD_t = -\rho D_t dt + \frac{1-\nu}{q} dN_t$.

Comments

- When fitted to market data, one may expect to find parameters different but not "too far" from to the MIHM case.
- $\beta = \rho$: the autocorrelation of trade signs is compensated by liquidity providers (same conclusion as in Bouchaud, Gefen, Potters and Wyart).
- $\alpha = (1-\nu)\beta$: when $\iota_c = 0$, α/β is the average number of child orders. $1-\nu$: proportion of transient (vanishing) impact.
- When $\varphi_s(x) = \iota_s$ and $\varphi_c(x) = \iota_c$, $\mu = \text{Dirac}(m_1)$ comes from the fact that all market orders have the same impact regardless of their size. $\rightarrow$ clustering of size orders around typical values.
- MIHM stationary iff $\iota_c < \nu\rho/2$.

Market stability

Definition 4

We say that a market admits weak Price Manipulation Strategies (wPMS) if the cost of a liquidation strategy can be reduced by trading immediately after other market orders.

Corollary 5

In the MIH model, the market does not admit wPMS if, and only if,

$$\beta = \rho,\ \alpha = (1-\nu)\rho \text{ and } \varphi_s(x) - \varphi_c(x) = \alpha x \text{ for } x \in \mathcal{S}(\mu),$$

or $\mu = \mathit{Dirac}(0)$.

In this case, $(\frac{m_1}{q}\delta_t - \rho D_t) = (\frac{m_1}{q}\delta_0 - \rho D_0)e^{-\rho t} \underset{t\to\infty}{\to} 0$.
Therefore excluding PMS or wPMS is quite equivalent in the MIH model.

The dataset

Data : time stamps of midprice changes, and at each time, the order type (market order, cancellation, etc.) and the queue size at the best bid and ask price.

Year	2012	2013	2012 – 2013
Average price	32.4	44.9	38.7
Tick size	0.005	0.005	0.005
First queue average size	1398	1136	1260
m_1	776	636	714
m_2/m_1^2	3.38	4.69	3.87
Midpoint changes per hour	1909	1699	1798
Prop. triggered by trades	10.0%	7.9%	9.0%

TABLE: Statistics for the stock BNP Paribas for the periods February-September 2012 and January-September 2013, between 11 a.m. and 1 p.m.

A Generalized Price model

General decay kernels $G, K : \mathbb{R}_+ \to \mathbb{R}_+$. We assume that the midprice follows

$$P_t = \sum_{\tau<t} \Delta M_\tau G(t-\tau) + \sigma W_t.$$

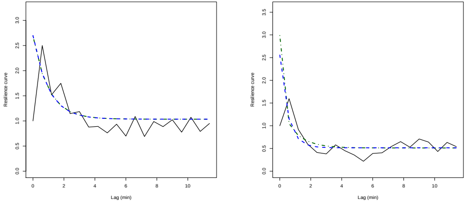
with

$$\kappa_t^+ = \kappa_\infty + \sum_{\tau<t} \left[\mathbb{1}_{\{\Delta N_\tau > 0\}} \varphi_s \left(\frac{\Delta N_\tau}{m_1} \right) + \mathbb{1}_{\{\Delta N_\tau < 0\}} \varphi_c \left(-\frac{\Delta N_\tau}{m_1} \right) \right] K(t-\tau),$$

$$\kappa_t^- = \kappa_\infty + \sum_{\tau<t} \left[\mathbb{1}_{\{\Delta N_\tau < 0\}} \varphi_s \left(-\frac{\Delta N_\tau}{m_1} \right) + \mathbb{1}_{\{\Delta N_\tau > 0\}} \varphi_c \left(\frac{\Delta N_\tau}{m_1} \right) \right] K(t-\tau).$$

$\Delta M_\tau, \Delta N_\tau$: midprice jumps and order size at time τ.
Key fact : Separate estimations of the price propagator (G and σ) and of the Hawkes process.

Calibration of the Price propagator G



(a) 2012 (b) 2013

FIGURE: BNP Paribas. The plain line is the unconstrained propagator, the (blue) dashed line is the mono-exponential resilience curve, and the (green) dot-dashed line is the multi-exponential resilience curve.

Estimation of the Hawkes parameters

Year	2012	2013
L_{adj} (sec)	4	2
γ_{multi}	2.69	2.99
ρ_{multi}	60	60/360
λ_{multi}	0.61	0.30/0.53
ν_{multi}	0.39	0.17
σ_{multi}	0.2253	0.2153
r^2_{multi}	24.677%	10.674%
γ_{mono}	2.70	2.56
ρ_{mono}	60.8	116.5
λ_{mono}	0.62	0.80
σ_{mono}	0.2253	0.2153
r^2_{mono}	24.678%	10.688%

$G(0) = 1$, G linear on $[0, L_{\text{adj}}]$ and for $t \geq L_{\text{adj}}$,
$G(t) = \gamma[\sum_{i=1}^{p} \lambda_i \exp(-\rho_i t) + 1 - \sum_{i=1}^{p} \lambda_i]$.

Year	2012	2013
β_{multi}	6/360	6/360
w_{multi}	0.010/0.990	0.011/0.989
$\kappa_{\infty\,\text{multi}}$	15.1	12.1
$\phi_{s\,\text{multi}}$	112.8/18.4	115.4/15.7
$\phi_{c\,\text{multi}}$	50.4/2.1	46.4/0.9
$\mathcal{L}_{\text{multi}}$	2.7720	2.6708
β_{mono}	73.0	114.1
$\kappa_{\infty\,\text{mono}}$	13.9	14.0
$\phi_{s\,\text{mono}}$	38.3/6.2	58.5/8.0
$\phi_{c\,\text{mono}}$	17.1/0.7	23.5/0.5
$\mathcal{L}_{\text{mono}}$	2.6826	2.5794

$K(t) = \sum_i w_i e^{-\beta_i t}$, $\sum_i w_i = 1$.

TABLE: Calibration of the resilience (left) and intensity (right) for the stock BNP Paribas for the periods February-September 2012 and January-September 2013, between 11 a.m. and 1 p.m. For the ϕ's, the first entry is the constant term and the second one is the linear term.

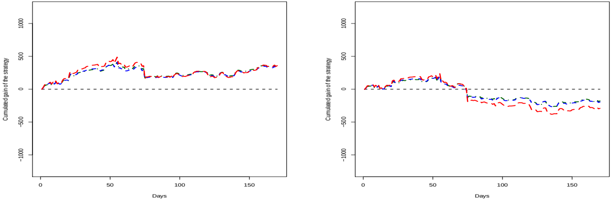
Backtest of the optimal strategy I

We run the (scaled) optimal strategy that liquidates 0 every day between 11 :30 and 13 :00.

Year	IS 2012	+bid-ask	IS 2013	+bid-ask	OS 2013	+bid-ask
Sharpe (Multi)	1.382	−0.675	2.454	0.725	2.248	0.418
Proba. (Multi)	65.9%	56.5%	61.3%	47.1%	58.1%	48.2%
Skew (Multi)	−2.02	−2.40	3.65	3.34	4.48	4.14
Kurtosis (Multi)	19.02	19.94	29.40	27.71	36.96	34.65
Sharpe (Mono)	1.263	−0.713	2.536	0.771	2.430	0.563
Proba. (Mono)	62.9%	57.1%	62.3%	48.2%	58.1%	49.7%
Skew (Mono)	−1.89	−2.30	2.94	2.61	3.56	3.21
Kurto. (Mono)	16.64	17.68	23.27	21.90	26.74	24.87

TABLE: The first two columns are In-Sample results, i.e. the data used to calibrate the model is the same as the evaluation data. The third column gives Out-of-Sample results, i.e. we calibrate the model on the 2012 data to apply the strategy on the 2013 data.

Backtest of the optimal strategy II



(a) Trading at the midprice (b) One half-tick penalty

FIGURE: Cumulated gains of the strategy applied on BNP Paribas on the period February-September 2012, every day between 11.30a.m. and 1p.m. Left : we allow the strategy to trade at the midprice. Right : we apply a posteriori a linear cost penalty of one half-tick to account for the bid-ask spread.

Conclusion

- The Mixed-market-impact Hawkes price model makes a bridge between OW model and higher frequency models.
- The optimal execution problem can be solved explicitly in this model.
- A particular parametrization of the Hawkes process allows to exclude PMS.
- The estimation of the model to market data can be achieved with some adjustments. A blind use of the optimal strategy do not give arbitrage.
- The calibrated model can be interesting for optimal execution to take into account the order flow. It might also give arbitrage in some market conditions.