PROBLEM CLASS: FFT AND QUADRATURE METHODS FOR OPTION PRICING

Let $(S_t)_{t\geq 0}$ denote the dynamics of the stock price process with initial value S_0 known. We are interested here in evaluating a European call option on S, with strike $K > 0$ and maturity $t > 0$. As we mentioned in the lecture notes, pricing a European option (call or put) boils down to integrating the density of the stock price agains the payoff of the option. Unfortunately, for most models the density is not available in closed-form and this approach is hence not possible. A different route to the pricing problem is pricing via characteristic functions, which turn out to be available in closed-form for large classes of models used in financial modelling. We turn our attention here to two specific models: the Heston stochastic volatility model, and an exponential Lévy model, namely the Variance-Gamma model. For notational convenience, we denote $X_t := \log(S_t)$ the logarithmic stock price process, and we define its characteristic function as

$$\phi_t(\xi) := \mathbb{E}\left(\mathrm{e}^{\mathtt{i}\xi X_t}\right), \qquad \text{for all } \xi \in \mathbb{R}.$$

In the Heston model, the process $(X_t)_{t\geq 0}$ satisfies the following stochastic differential equation:

$$\begin{aligned} \mathrm{d}X_t &= -\frac{1}{2}V_t\mathrm{d}t + \sqrt{V_t}\mathrm{d}W_t, \\ \mathrm{d}V_t &= \kappa\left(\theta - V_t\right)\mathrm{d}t + \sigma\sqrt{V_t}\mathrm{d}Z_t, \end{aligned}$$

where W and Z are two Brownian motions correlated with $\rho \in (-1, 1)$. $\kappa > 0$ is the mean-reversion strength, $\theta > 0$ the long-term variance and $\sigma > 0$ the volatility of variance. The initial level of the instantaneous variance $V_0 = v_0 > 0$ is known, and we normalise the process so that $X_0 = 0$. We have

$$\phi_t^{\mathrm{H}}(\xi) = \exp\Big(C_t(\xi) + D_t(\xi)v_0\Big),$$

where

$$\begin{aligned} C_t(\xi) &:= \frac{\kappa\theta}{\sigma^2}\left(\left(\kappa - \mathtt{i}\rho\sigma\xi + d(\xi)\right)t - 2\log\left(\frac{\gamma(\xi)\mathrm{e}^{d(\xi)t} - 1}{\gamma(\xi) - 1}\right)\right), \\ D_t(\xi) &:= \frac{\kappa - \mathtt{i}\rho\sigma\xi + d(\xi)}{\sigma^2}\left(\frac{\mathrm{e}^{d(\xi)t} - 1}{\gamma(\xi)\mathrm{e}^{d(\xi)t} - 1}\right), \\ d(\xi) &:= \sqrt{\left(\kappa - \mathtt{i}\rho\sigma\xi\right)^2 + \sigma^2\left(\xi^2 + \mathtt{i}\xi\right)}, \qquad \gamma(\xi) := \frac{\kappa - \mathtt{i}\rho\sigma\xi + d(\xi)}{\kappa - \mathtt{i}\rho\sigma\xi - d(\xi)} \end{aligned}$$

The Variance-Gamma is a Lévy model with no continuous paths, i.e. is a pure jump process. Its characteristic function reads

$$\phi_t^{\mathrm{VG}}(\xi) = \left(1 - \mathtt{i}\theta\nu\xi + \frac{\sigma^2\nu}{2}\xi^2\right)^{-t/\nu},$$

where $\nu > 0$, $\sigma > 0$ and $\theta \in \mathbb{R}$.

The goal of the problem class is to use the Carr-Madan formula (with the dampening factor α) to price a European call option with the following three methods:

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(i) Adaptive Gauss-Legendre quadrature.

(ii) Fast Fourier Transforms with Simpson's rule.

(ii) Fractional Fast Fourier Transforms with Simpson's rule.

Additionally, for a fixed time t, compute the density of each stock price at time t.