interpolationPrices

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Contents

0.1 Linear and quadratic interpolation of option prices

We illustrate here the influence of linear/quadratic interpolation of (European) option prices on the shape of the corresponding implied volatility smile.

0.1.1 Black-Scholes functions

```
In [1]: from math import log, sqrt, exp
        from scipy.stats import norm
        from scipy.optimize import bisect
        from zanadu.Groups.ImperialCollege.Root.Tools.BlackScholes import BlackScholes
        from zanadu.Groups.ImperialCollege.Root.Tools.ImpliedVolLeeLi import impliedVolCore
```

0.1.2 SVI parameterisation

We wish to study here the influence, on the implied volatility smile, of linear and quadratic interpolation (in strike) of option prices. We generate a fixed number of option prices, for a given maturity, using the SVI parameterisation:

$$\mathrm{SVI}(x) = a + b\left\{\rho(x-m) + \sqrt{(x-m)^2+\sigma^2}\right\}.$$

```
In [2]: def SVI(sviParams, x):
            ########### SVI parameterisation for the implied volatility sm
             # We return the square root!!!!!!!!!!!!!!!!!!!!!!!!!!!
            a, b, rho, m, sigma = sviParams
            return sqrt(a + b * (rho * (x - m) + sqrt((x - m) * (x - m) + sigma * sigma)))
```

0.1.3 Interpolation of option prices

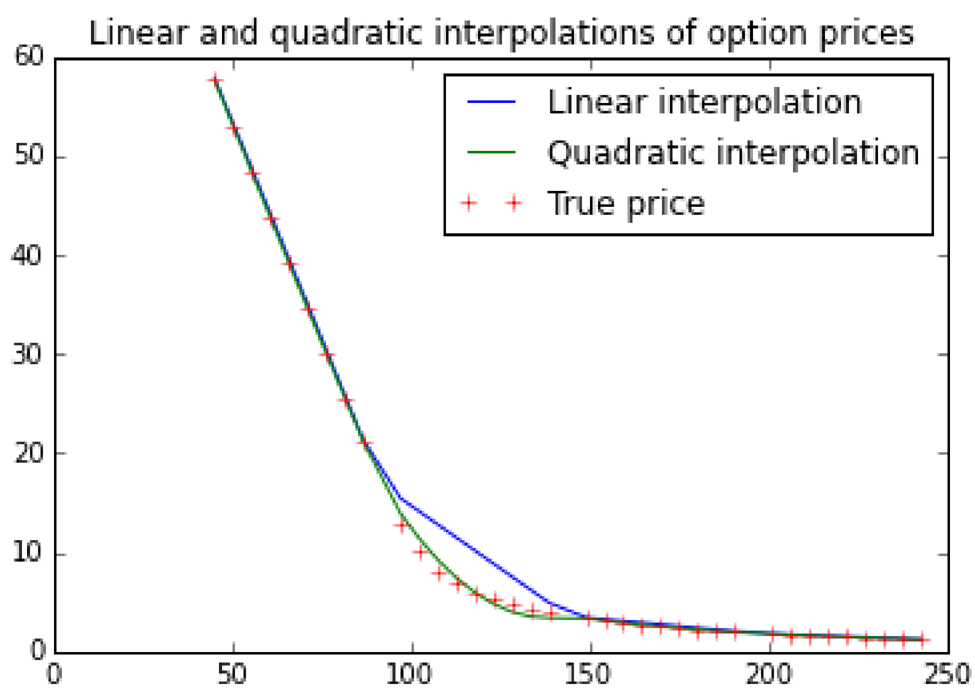
```
In [4]: def interpolationPricesVols(S, v, T, n, nb, minVal, maxVal, sviParams):

            prices, strikes, myStrikes, vols, volsSVI, pricesSVI, volsLin, volsQuad, pricesQuad, pricesL
            ], [], [], [], [], [], [], [], [], []
            counter = 1

            #################  Initialisation  ########################
            X = minVal
            strikes.append(X)
            vol = SVI(sviParams, log(X / S))
            P = BlackScholes(True, S, X, T, 0.0, 0.0, vol)
            prices.append(P)
            vols.append(vol)
```

```python
##################   Loop   ##########################

for i in range(1, n):
   X = minVal + 1.0 * i * (maxVal - minVal) / (1.0 * n)
   strikes.append(X)
   vol = SVI(sviParams, log(X / S))
   P = BlackScholes(True, S, X, T, 0.0, 0.0, vol)
   prices.append(P)
   vols.append(vol)

   if (i % 2 == 0):
      a = (prices[i] - prices[i - 2]) / \
          (strikes[i] - strikes[i - 2])
      b = prices[i] - a * strikes[i]

      for j in range(1, nb):
         X = strikes[i - 2] + 1.0 * j * \
             (strikes[i] - strikes[i - 2]) / (1.0 * nb)
         P = a * X + b
         myStrikes.append(X)
         myVol = impliedVolCore(
             True, 1.0, S, X, T, P, tolerance=1e-6, itermax=100)
         pricesLin.append(P)
         volsLin.append(myVol)

         # Price by quadratic interpolation
         P = prices[i - 2] * (X - strikes[i - 1]) * (X - strikes[i]) \
             / ((strikes[i - 2] - strikes[i - 1]) * (strikes[i - 2] - strikes[i])) \
             + prices[i - 1] * (X - strikes[i - 2]) * (X - strikes[i]) \
             / ((strikes[i - 1] - strikes[i - 2]) * (strikes[i - 1] - strikes[i])) \
             + prices[i] * (X - strikes[i - 2]) * (X - strikes[i - 1]) \
             / ((strikes[i] - strikes[i - 2]) * (strikes[i] - strikes[i - 1]))

         myVol = impliedVolCore(
             True, 1.0, S, X, T, P, tolerance=1e-6, itermax=100)

         pricesQuad.append(P)
         volsQuad.append(myVol)
         volsSVI.append(SVI(sviParams, log(X / S)))
         pricesSVI.append(
             BlackScholes(True, S, X, T, 0.0, 0.0, SVI(sviParams, log(X / S))))
return myStrikes, pricesLin, pricesQuad, pricesSVI, volsLin, volsQuad, volsSVI
```

0.1.4 Numerical tests

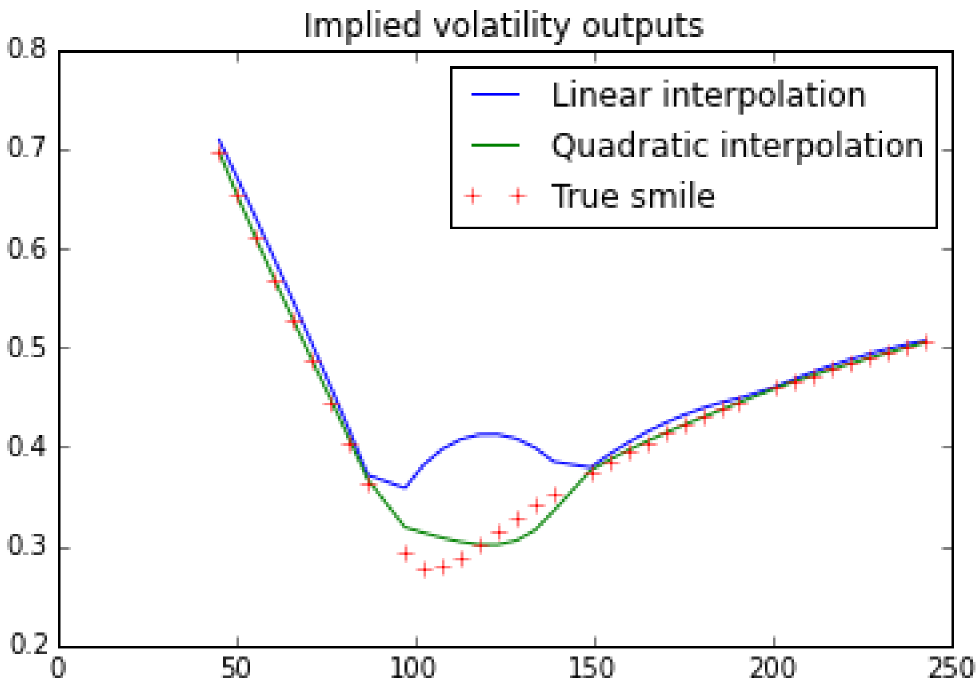
```
In [7]: S, v, T = 100., 0.2, 1.
        n = 10  # has to be an even integer
        nb = 10  # number of interpolated points between two strikes

        minVal, maxVal = 40., 300.
        #################SVI parameters################
        sviParams = 0.04, 0.4, -0.4, 0., 0.1

        strikes, pricesLin, pricesQuad, pricesSVI, volsLin, volsQuad, volsSVI = interpolationPricesVols
            S, v, T, n, nb, minVal, maxVal, sviParams)
```

```
In [9]: plot(strikes, pricesLin, 'b', label="Linear interpolation")
        plot(strikes, pricesQuad, 'g', label="Quadratic interpolation")
        plot(strikes, pricesSVI, 'r+', label="True price")
        legend(loc=1)
        title("Linear and quadratic interpolations of option prices")
        show()
```




```
In [10]: plot(strikes, volsLin, 'b', label="Linear interpolation")
         plot(strikes, volsQuad, 'g', label="Quadratic interpolation")
```

```
plot(strikes, volsSVI, 'r+', label="True smile")
legend(loc=1)
title("Implied volatility outputs")
show()
```



In [49]: