FiniteDiffFirstDeriv

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Contents

1 Finite difference approximations for first derivatives

```
In [6]: import pylab as plt
        import numpy as np
```

We consider the function $g(x) = \text{atan}(x)$, the derivatives of which reads $g'(x) = \frac{1}{1+x^2}$.

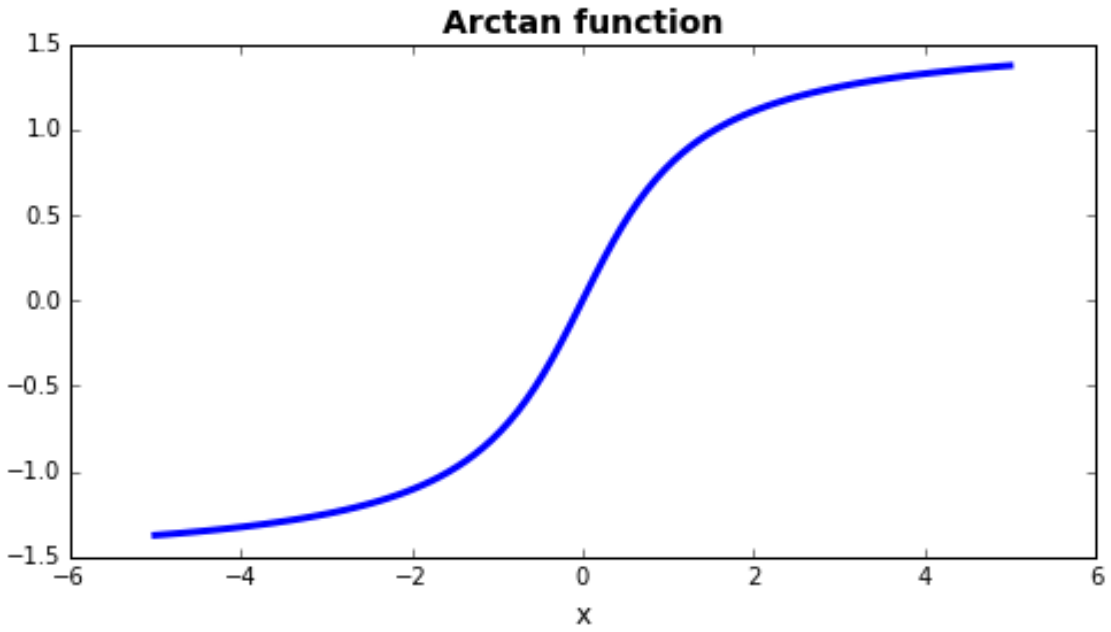
```
In [7]: def g(x):
            return np.arctan(x)


        def g_diff(x):
            return 1. / (1. + x * x)


        def g_diff_left(x, eps):
            return (g(x + eps) - g(x)) / eps


        def g_diff_right(x, eps):
            return (g(x) - g(x - eps)) / eps


        def g_diff_central(x, eps):
            return (g(x + eps) - g(x - eps)) / (2. * eps)
```

1.1 Numerical tests

1.1.1 As a function of x, for fixed ϵ.

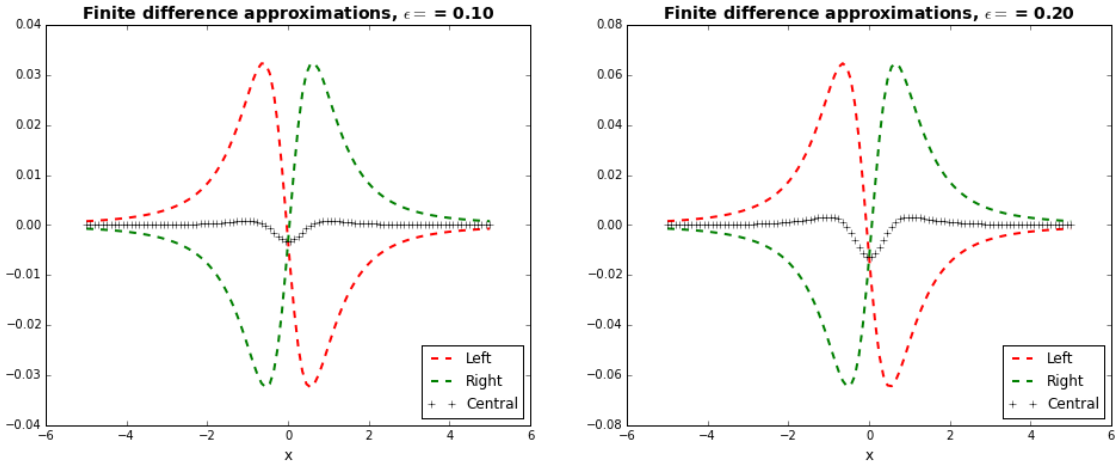
```
In [10]: xx = linspace(-5., 5., 100)
         eps = [0.1, 0.2]
```

```
In [11]: plt.figure(figsize=(8, 4))
         plt.plot(xx, [g(x) for x in xx], 'b', linewidth=3)
         plt.title("Arctan function", fontsize=14, fontweight='bold')
         plt.xlabel(u'x', fontsize=12)
         plt.show()
```




```
In [12]: DTrue = [g_diff(x) for x in xx]
         DLeft = [[g_diff_left(x, e) for x in xx] for e in eps]
         DRight = [[g_diff_right(x, e) for x in xx] for e in eps]
         DCentral = [[g_diff_central(x, e) for x in xx] for e in eps]

         ErrorsLeft = [[dA - dtrue for (dA, dtrue) in zip(DLeft[0], DTrue)], [
             dA - dtrue for (dA, dtrue) in zip(DLeft[1], DTrue)]]
         ErrorsRight = [[dA - dtrue for (dA, dtrue) in zip(DRight[0], DTrue)], [
             dA - dtrue for (dA, dtrue) in zip(DRight[1], DTrue)]]
         ErrorsCentral = [[dA - dtrue for (dA, dtrue) in zip(DCentral[0], DTrue)], [
             dA - dtrue for (dA, dtrue) in zip(DCentral[1], DTrue)]]

         plt.figure(figsize=(16, 6))
         plt.subplot(121)
         plt.plot(xx, ErrorsLeft[0], 'r--', linewidth=2, label="Left")
         plt.plot(xx, ErrorsRight[0], 'g--', linewidth=2, label="Right")
         plt.plot(
             xx, ErrorsCentral[0], 'k+', linewidth=1, label="Central")
         plt.title('Finite difference approximations, $\epsilon=$ = %.2f' %
                   eps[0], fontsize=14, fontweight='bold')
         plt.xlabel(u'x', fontsize=12)
         plt.legend(loc=4)
         plt.subplot(122)
         plt.plot(xx, ErrorsLeft[1], 'r--', linewidth=2, label="Left")
```

```
    plt.plot(xx, ErrorsRight[1], 'g--', linewidth=2, label="Right")
    plt.plot(
        xx, ErrorsCentral[1], 'k+', linewidth=1, label="Central")
    plt.title('Finite difference approximations, $\epsilon=$ = %.2f' %
              eps[1], fontsize=14, fontweight='bold')
    plt.xlabel(u'x', fontsize=12)
    plt.legend(loc=4)
    plt.show()
```

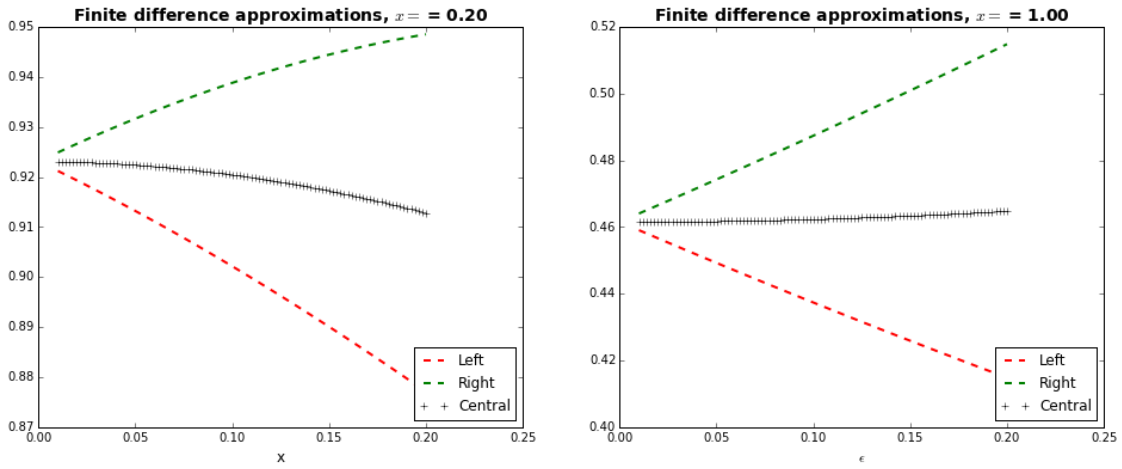


1.1.2 As a function of ϵ, for fixed x.

In [13]:
```
eps = linspace(1E-2, 0.2, 100)
xx = [0.2, 1.]
```

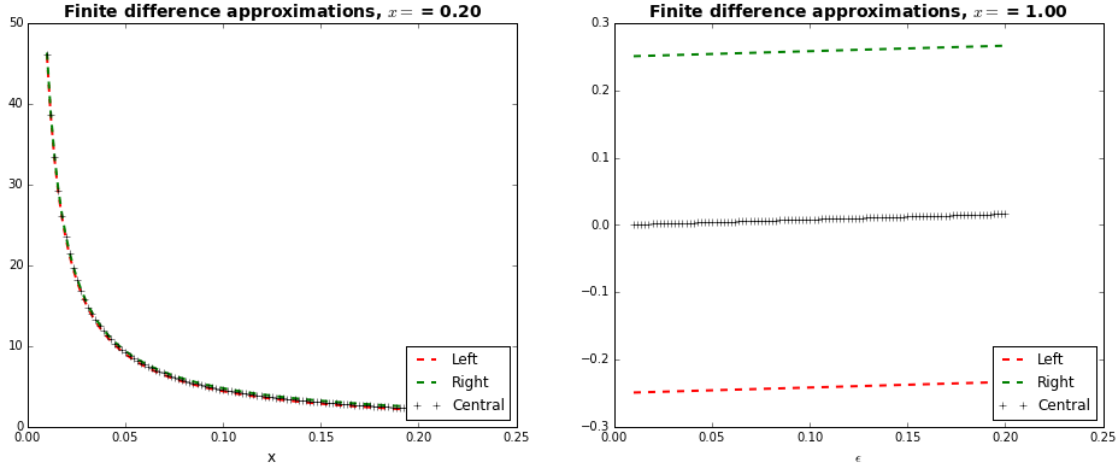
In [14]:
```
DTrue = [g_diff(x) for e in eps]
DLeft = [[g_diff_left(x, e) for e in eps] for x in xx]
DRight = [[g_diff_right(x, e) for e in eps] for x in xx]
DCentral = [[g_diff_central(x, e) for e in eps] for x in xx]

ErrorsLeft = [[dA - dtrue for (dA, dtrue) in zip(DLeft[0], DTrue)], [
    dA - dtrue for (dA, dtrue) in zip(DLeft[1], DTrue)]]
ErrorsRight = [[dA - dtrue for (dA, dtrue) in zip(DRight[0], DTrue)], [
    dA - dtrue for (dA, dtrue) in zip(DRight[1], DTrue)]]
ErrorsCentral = [[dA - dtrue for (dA, dtrue) in zip(DCentral[0], DTrue)], [
    dA - dtrue for (dA, dtrue) in zip(DCentral[1], DTrue)]]

plt.figure(figsize=(16, 6))
plt.subplot(121)
```

```
    plt.plot(eps, ErrorsLeft[0], 'r--', linewidth=2, label="Left")
    plt.plot(eps, ErrorsRight[0], 'g--', linewidth=2, label="Right")
    plt.plot(
        eps, ErrorsCentral[0], 'k+', linewidth=1, label="Central")
    plt.title('Finite difference approximations, $x=$ %.2f' %
              xx[0], fontsize=14, fontweight='bold')
    plt.xlabel(u'x', fontsize=12)
    plt.legend(loc=4)
    plt.subplot(122)
    plt.plot(eps, ErrorsLeft[1], 'r--', linewidth=2, label="Left")
    plt.plot(eps, ErrorsRight[1], 'g--', linewidth=2, label="Right")
    plt.plot(
        eps, ErrorsCentral[1], 'k+', linewidth=1, label="Central")
    plt.title('Finite difference approximations, $x=$ %.2f' %
              xx[1], fontsize=14, fontweight='bold')
    plt.xlabel(u'$\epsilon$', fontsize=12)
    plt.legend(loc=4)
    plt.show()
```




```
In [54]: DTrue = [g_diff(x) for e in eps]
         DLeft = [[g_diff_left(x, e) for e in eps] for x in xx]
         DRight = [[g_diff_right(x, e) for e in eps] for x in xx]
         DCentral = [[g_diff_central(x, e) for e in eps] for x in xx]

         ErrorsLeft = [[(dA - dtrue) / e for (dA, dtrue, e) in zip(DLeft[0], DTrue, eps)],
                       [(dA - dtrue) / e for (dA, dtrue, e) in zip(DLeft[1], DTrue, eps)]]
         ErrorsRight = [[(dA - dtrue) / e for (dA, dtrue, e) in zip(DRight[0], DTrue, eps)],
                        [(dA - dtrue) / e for (dA, dtrue, e) in zip(DRight[1], DTrue, eps)]]
```

```
ErrorsCentral = [[(dA - dtrue) / e for (dA, dtrue, e) in zip(DCentral[0], DTrue, eps)],
                 [(dA - dtrue) / e for (dA, dtrue, e) in zip(DCentral[1], DTrue, eps)]]

plt.figure(figsize=(16, 6))
plt.subplot(121)
plt.plot(eps, ErrorsLeft[0], 'r--', linewidth=2, label="Left")
plt.plot(eps, ErrorsRight[0], 'g--', linewidth=2, label="Right")
plt.plot(
    eps, ErrorsCentral[0], 'k+', linewidth=1, label="Central")
plt.title('Finite difference approximations, $x=$ = %.2f' %
          xx[0], fontsize=14, fontweight='bold')
plt.xlabel(u'x', fontsize=12)
plt.legend(loc=4)
plt.subplot(122)
plt.plot(eps, ErrorsLeft[1], 'r--', linewidth=2, label="Left")
plt.plot(eps, ErrorsRight[1], 'g--', linewidth=2, label="Right")
plt.plot(
    eps, ErrorsCentral[1], 'k+', linewidth=1, label="Central")
plt.title('Finite difference approximations, $x=$ = %.2f' %
          xx[1], fontsize=14, fontweight='bold')
plt.xlabel(u'$\epsilon$', fontsize=12)
plt.legend(loc=4)
plt.show()
```



In [ ]: