## **Biomedical Engineering Year 1**

2.

## Mathematics Problems 8

1. In each of the following cases, find whether the integral exists, and if so find its value.

(i) 
$$\int_{3}^{\infty} x^{-3/2} dx$$
.  
(ii)  $\int_{1}^{2} (x-1)^{-2/3} dx$ .  
(iii)  $\int_{0}^{1} \ln x dx$ .  
(iii)  $\int_{0}^{1} \ln x dx$ .  
Evaluate  
(i)  $\int_{-2}^{2} \sinh(x^{3}) dx$ ,  
(ii)  $\int_{-3}^{3} e^{-5|x|} dx$ .

3. Determine the length of the curve  $y = \cosh x$  which lies between the lines x = 0 and x = 1. (This curve, called a catenary, is the shape of a heavy cable hanging under gravity.)

4. Find the length of the curve 
$$y = \frac{x^2}{4} - \frac{1}{2} \ln x$$
 between  $x = 1$  and  $x = 3$ .

5. The curve described by the cable of a suspension bridge is given by

$$y = \frac{H}{L^2} x^2 - 2 \frac{H}{L} x + H$$

where x is the distance from one end of the bridge of length 2L and H is the height of the top of the cable above its lowest point. Show that the length of the cable is

$$(L^2 + 4H^2)^{1/2} + \frac{1}{2} \frac{L^2}{H} \sinh^{-1} \frac{2H}{L}$$

If *H* is much smaller than *L*, show that the length of the cable is approximately  $2L\left\{1 + \frac{2}{3}\left(\frac{H}{L}\right)^2\right\}$ . [Hint: put  $\varepsilon = \frac{2H}{L}$  and note that  $\sinh^{-1}\varepsilon = \varepsilon - \frac{\varepsilon^3}{6} + \dots$ ] 6. Find the length of the curve  $x = \theta + \sin \theta$ ,  $y = 1 + \cos \theta$  between  $\theta = 0$  and  $\theta = \pi$ .

## Answers for Problems 7