Biomedical Engineering Year 1 **Mathematics Problems 10**

1. Find the general solutions of the following equations.

   (a) $y'' + 3y' - 4y = 7\,e^{3x}$.

   (b) $y'' - 3y' + 2y = e^{2x}$.

   (c) $y'' - 4y' + 4y = e^{2x}$.

   (d) $2y'' - 3y' + y = x^2$.

   (e) $y'' - 2y' + 2y = e^x \sin x$.

   (f) $y'' + y' + y = 49xe^{2x}$.

   (g) $y'' - 2y' + y = x^3$.

   (h) $y'' + 9y = \cos^2 x$.

2. Find the solution of the equation

$$y'' + 6y' + 8y = 12\cosh 2x\,,$$

   for which $y(0) = 0$ and $y'(0) = 1$.

3. Show that the substitution $x = e^t$ converts the equation

$$ax^2\tfrac{d^2y}{dx^2} + bx\tfrac{dy}{dx} + cy = 0\,,$$

   in which a, b and c are constants, into an equation with constant coefficients.
   Hence find the general solution of the equation

$$x^2\tfrac{d^2y}{dx^2} + x\tfrac{dy}{dx} + y = 0\,.$$

4. A particle of mass m moves along the x axis under the influence of a restoring force $-m\omega^2 x$ and a frictional force mk, where ω and k are positive constants. The frictional force acts at all times to oppose the motion. If the particle is launched from the origin with velocity V in the positive x direction, show that it first comes to rest at time $t = T$, where $\omega T = \tan^{-1}(\omega V/k)$ and find $x(T)$.

   (Optional) Show that no further motion can take place unless $V > 3^{1/2}k/\omega$.
   [Hint : for $t > T$, the equation of motion of the particle is $mx'' = -m\omega^2 x + mk$, and $x'(T) = 0$. Find $x(t)$ for $t > T$ and consider $x'(t)$ for $t > T$.]

5. A raindrop of mass m falling through the atmosphere is subject to air resistance proportional to the square of its velocity v. Show that the equation of motion is

$$m\frac{dv}{dt} = mg - mkv^2$$

   where g is the acceleration due to gravity and k is a positive constant. If x is the distance fallen after time t from an initial state of rest, show that $\dfrac{dy}{dx} = 2(g - ky)$, where $y = v^2$.
   Find v as a function of x and determine the terminal velocity.
   [Hint : $\dfrac{dv}{dt} = \dfrac{dv}{dx}\dfrac{dx}{dt} = \dfrac{dv}{dx}\,v = \dfrac{1}{2}\dfrac{d}{dx}\,v^2$.]

P.T.O.

Answers for Problems 9

1. (i) $y' = y\cot x + \sin x\,.$ (ii) $x^2y' - xy = y^2\,.$
2. (i) $y = 2x + 1\,.$ (ii) $y = 1/2(1 + x^2)^2\,.$ (iii) $y = 1 \pm (1 - x^2)^{\frac{1}{2}}\,.$ (iv) $y = 2\,x\,\left(1 - k\,x^2\right)^{-1}\,.$ (v) $xy^2 + x^3 = c\,.$ (vi) $x^3 + y^3 = cxy\,.$ (vii) $y = (1 - 2\,\cos x)\cos x\,.$ (viii) $y = (2x + c)\,e^{(-x^2)}\,.$ (ix) $y = \frac{1}{4}x^2 - \frac{1}{3}x + \frac{1}{2} - \frac{1}{12}x^{-2}\,.$
3. $A = 2$ and $B = -1.$ $(x - y - 3)^4 = k\,(2x + y - 3).$
4. (i) $3x^3 + xy - x - 2y^2 = c\,.$ (ii) $x^2y^2 + 2xy = c\,.$
5. $n = 2.$ $x^5 + 3x^4y - x^3y^2 = c.$
6. $xy^4 - 2xy^2 + \frac{1}{2}x^2y^6 - x^2y^4 + \frac{1}{2}y^2 = c.$
7. $\dfrac{2}{5}\,\dfrac{R^2H^{1/2}}{ka^2}\,.$