

# Algebraic number theory

## Test 2

18 March, 2011

*You can use all statements from lectures or problem sheets without proof.*

1. List all quadratic fields ramified exactly at the prime 2011. (A very brief explanation will suffice.)
2. Let  $K$  be a quadratic field, and  $I \subset \mathcal{O}_K$  be an ideal of norm  $n$ . Let  $\bar{I} = \{\bar{z} | z \in I\}$ , where bar denotes conjugation. Prove that the fractional ideal  $\frac{1}{n}\bar{I} \subset K$  is the inverse of  $I$ . (A few sentences will suffice.)
3. Let  $K = \mathbb{Q}(\sqrt{-5})$ . Write the principal ideal  $(7 + \sqrt{-5}) \subset \mathcal{O}_K$  as the product of prime ideals.
4. Let  $K = \mathbb{Q}(\sqrt[3]{2})$ . It is known that  $\mathcal{O}_K = \mathbb{Z} \oplus \mathbb{Z}\sqrt[3]{2} \oplus \mathbb{Z}(\sqrt[3]{2})^2$ . Using this fact compute the discriminant of  $K$ . (A very brief explanation will suffice.)