**Exercises for all the project**

1. Consider $A = \begin{pmatrix} 2 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix}$ and the corresponding replicator system

$$\dot{x}_i = x_i((Ax)_i - x \cdot Ax). \qquad (*)$$

   Determine the NE and ESS points of this system. Draw the indifference lines, and best response regions in the appropriate triangle $\Delta$. Determine the singularity points of (*), and determine the eigenvalues at these singularities, and thus whether they are saddle points, attracting or repelling. Sketch the phase portrait of (*). You are allowed to use theorems in the lecture notes.

2. Consider the

$$\text{battle of the sexes game: } \begin{pmatrix} (2,1) & (0,0) \\ (0,0) & (2,7) \end{pmatrix}$$

   (where we use the 2nd notation). Compute the NE, CCE and CE sets.

3. We say that two $m \times n$ bimatrix games $(A, B)$ and $(\tilde{A}, \tilde{B})$ are *(linearly) equivalent*, $(A, B) \sim (\tilde{A}, \tilde{B})$, if there exist $c, d > 0$ and $c_1, \ldots, c_n, d_1, \ldots, d_m \in \mathbb{R}$ so that

$$\tilde{a}_{ij} = c \cdot a_{ij} + c_j \quad \text{and} \quad \tilde{b}_{ij} = d \cdot b_{ij} + d_i \qquad \text{for } i = 1, \ldots, m \text{ and } j = 1, \ldots, n.$$

   Here we use the 2nd notation for bimatrix games. Show that if $(A, B)$ and $(\tilde{A}, \tilde{B})$ are equivalent then

   (a) the replicator dynamics associated to $(A, B)$ and to $(\tilde{A}, \tilde{B})$ are the same (or do you need $c = d$ for this?);

   (b) $\mathcal{B}R_A = \mathcal{B}R_{\tilde{A}}$, $\mathcal{B}R_B = \mathcal{B}R_{\tilde{B}}$ and the best response dynamics associated to $(A, B)$ and to $(\tilde{A}, \tilde{B})$ are the same.

   (c) the NE, CE and CCE sets associated to $(A, B)$ and to $(\tilde{A}, \tilde{B})$ are the same.

   (d) the Hart and Mas-Colell regret matching algorithms for these games give the same dynamics.

   (e) explain why the reinforcement learning algorithms from Section 6.1 of the lecture notes are not necessarily the same for the games $(A, B)$ and to $(\tilde{A}, \tilde{B})$.