Dynamics of learning and iterated games 2021-2022
Project 3: Regret minimisation learning, a computer implementation

In this project you are allowed to use books and the hand-outs, but it is important that you write up your project by yourself and that you can explain what you have written in detail during the short oral.

This project considers regret learning in the context of the Blotto games, which often is associated to two candidates for an election, who each can allocate money in different states. For a useful description see

- `https://www.wikiwand.com/en/Blotto_game`
- `https://en.wikipedia.org/wiki/Blotto_game`.

Attached to this this project description is a paper by

- T. W. Neller and Marc Lanctot, *An Introduction to Counterfactual Regret Minimization*,

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1. Submit your solutions of the selected exercises from the lecture notes (which are the same for all projects).
2. Explain the captain Blotto game described in the paper by Neller and Lanctot.
3. This game is not described in matrix form. Outline how many actions each player has, and how *in principle* to convert this game into game in matrix form.
4. Explain why this game must have a Nash equilibrium if the allocation to each battle field is an integer.
5. Describe the set of NE for one of the following cases:

   [-] in the case when there are $N = 2$ battle fields with not necessarily equal 'armies' and the allocation to each battle field does not need to be an integer. For this you can consult S. T. Macdonell and N. Mastronardi, *Waging simple wars: a complete characterization of two-battlefield Blotto equilibria*, Economic Theory, 2015) 58:183–216. `https://scottmacdonell.files.wordpress.com/2014/04/wagingsimplewars.pdf`. Explain what the notion of Nash equilibrium means in this case with an 'infinite' number of strategies.

   [-] in the corresponding case with equal 'armies' when there are $N = 3$ battle fields and again the allocation to each battle field does not need to be an integer. For this you can consult D. Kvasov, *Contests with limited resources*, Journal of Economic Theory 136 (2007) 738 – 748, `https://www.sciencedirect.com/science/article/pii/S0022053106001244`. Explain what the notion of Nash equilibrium means in this case with an 'infinite' number of strategies.

[-] assuming that the allocation to each battle field is an integer. For this you can consult S. Hart, *Discrete Colonel Blotto and General Lotto games*, Internat. J. Game Theory 36 (2008), no. 3-4, 441–460. `http://www.ma.huji.ac.il/hart/abs/blotto.html`

6. Outline the idea the regret learning algorithm, and explain to what set this algorithm is guaranteed to converge.

7. Implement regret learning to solve Exercise 2.6 in the paper by Neller and Lanctot. Also address the two following issues: Does you algorithm converge and to what set/point? If it converges, with what speed?

8. (**Mastery question for 4th year and MSc students**) Explain the theoretical arguments behind the regret learning algorithm (discussing for example speed of convergence). Also code up a reinforcement learning algorithm of your choice for this game. Compare the results you get with what you discussed in question 5.