

Identifying Self-Organised Criticality in nature

A guide by the confused

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Outline

- 1 Definition of SOC
- 2 The (field) theory of SOC
- 3 Observables and Analysis
- 4 Summary

What is SOC?

As far as general use goes, what does SOC normally refer to?

Two extremes:

- Anything where “critical behaviour” is observed without tuning of a parameter.
- Anything avalanching.

What is SOC?

Critical behaviour without tuning?

Typical criticism:

- Is the Ising Model at $T = T_c$ SOC?
- Is percolation SOC ($p_c = 1/2$ for square, bond and triangular, site)?
- Is a fair random walker SOC?
- Is a fair branching process SOC?
- Is turbulence SOC?

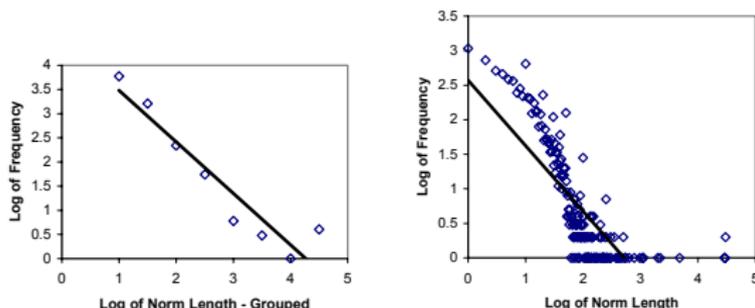
What is SOC?

A footnote on turbulence

- Scaling largely a matter of dimensional analysis (trivial?)
- Separation of time scales in “output” rather than driving (Grinstein, 1995)
- Flow of energy to *smaller and smaller* length scales.
- Definition of avalanches only via explicit thresholding (not those of the dynamics)

What is SOC?

Anything avalanching?

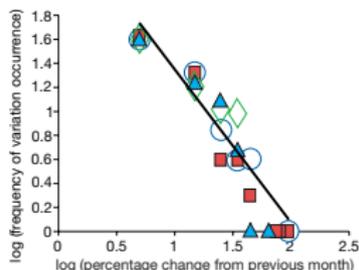


Hoffmann, 2005, Figs. 9 and 10

- Wars (Roberts & Turcotte, 1998)
- Pop charts (Bentley & Maschner, 1999)
- Urban Development (Batty & Xie, 1999)
- Hospital waiting times (Smethurst & Williams, 2001)
- Avalanches of social norms (Hoffmann, 2005)

What is SOC?

Anything avalanching?



Smethurst & Williams, 2001, Fig. 1

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What is SOC?

Anything with a power law ?

- Gravity, $F \propto r^{-2}$
- Hospital waiting times (Smethurst & Williams, 2001)
- Percolation

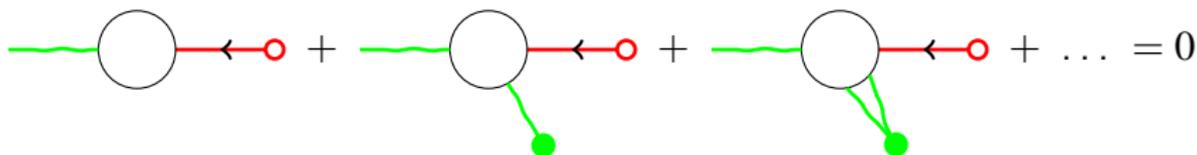
SOC

- 1 Non-trivial Scaling (finite size scaling — no control parameter)
- 2 Spatio-temporal correlations
- 3 Apparent self-tuning (underlying 2nd order phase transition?)
- 4 Separation of time scales
- 5 Avalanching (intermittency)
- 6 [nonlinear (thresholds) interaction] (supposedly required by 1)

SOC: Non-trivial scale invariance (spatio-temporal correlations!) in avalanching (intermittent) systems as known from ordinary critical phenomena, but with internal, self-organised rather than external tuning of a control parameter (to a non-trivial value).

The (field) theory of SOC

Stationarity is equivalent to self-organisation to critical point.
Stationarity (lack of additional net deposition):



- Vanishing deposition at stationarity means that the diagrams in the bracket vanish .
- Requires adjustment of **substrate** .
- Independent of **driving** .

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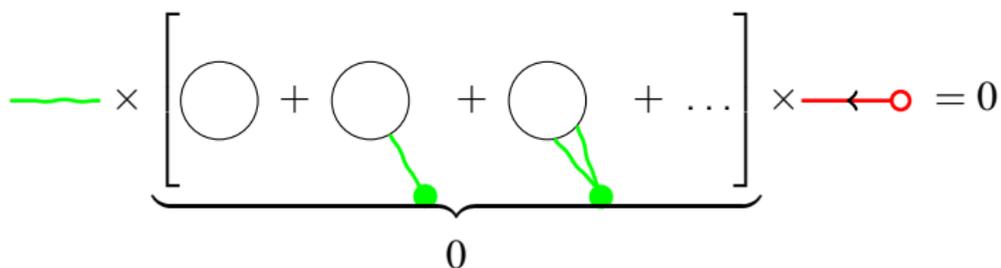
$$\text{---} \times \left[\bigcirc + \bigcirc \begin{array}{l} \text{---} \\ \bullet \end{array} + \bigcirc \begin{array}{l} \text{---} \\ \text{---} \\ \bullet \end{array} + \dots \right] \times \text{---} \leftarrow \bigcirc = 0$$

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The (field) theory of SOC

Stationarity is equivalent to self-organisation to critical point.

Stationarity (lack of additional net deposition):



The diagram shows a mathematical equation where a green wavy line is multiplied by a bracketed sum of circles. The first circle is empty. The second circle has a green dot at its bottom with a green line connecting it to the wavy line. The third circle has two green dots at its bottom with two green lines connecting them to the wavy line. This is followed by an ellipsis. The bracketed sum is multiplied by a red arrow pointing left towards a red circle. The entire expression is set equal to zero. A large zero is placed below the bracketed sum.

$$\text{---} \times \left[\bigcirc + \bigcirc + \bigcirc + \dots \right] \times \leftarrow \bigcirc = 0$$

0

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The (field) theory of SOC

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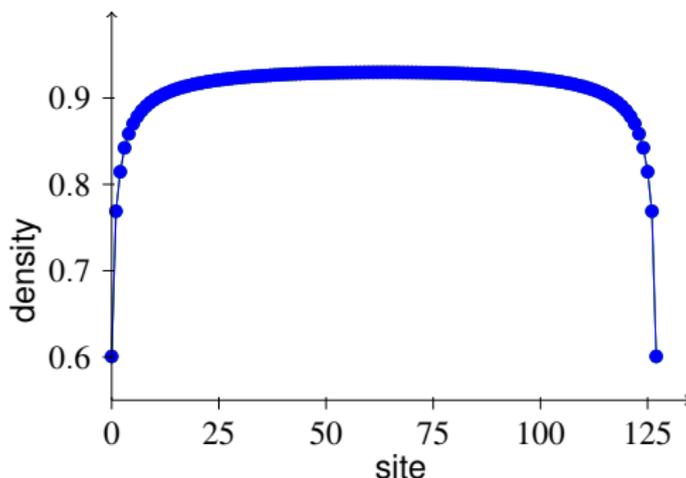
Stationarity (lack of additional net deposition):

The diagram shows an equation: a green wavy line on the left is multiplied by a large square bracket containing a series of circles. The first circle is empty. The second circle has a green dot at its bottom, with a green line extending downwards. The third circle has a green dot at its bottom, with two green lines extending downwards. This is followed by a plus sign and an ellipsis. To the right of the bracket is a red circle with a red arrow pointing left towards it, followed by an equals sign and a zero. A blue dashed arrow points from the text 'Vanishing deposition' to the zero. A green dashed arrow points from the text 'adjustment of substrate' to the green dots. A red dashed arrow points from the text 'Independent of driving' to the red circle.

$$\text{---} \times \left[\bigcirc + \bigcirc + \bigcirc + \dots \right] \times \leftarrow \bigcirc = 0$$

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At or around criticality

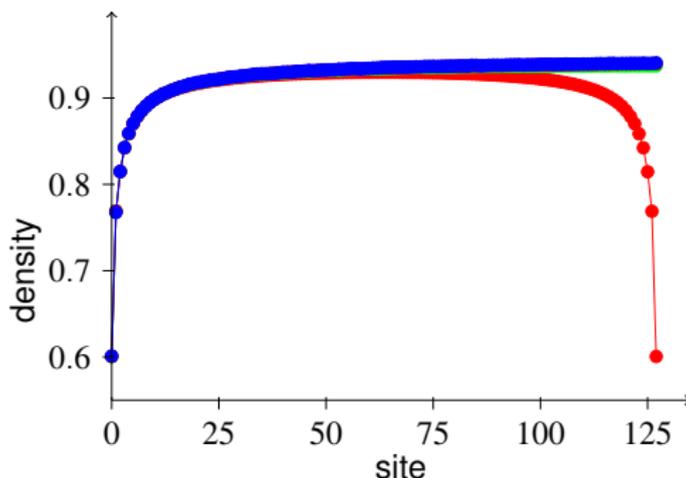


Driving **uniformly**, **at site 1**, **at site 0**.

The Manna Model is **at** criticality: No hovering, no sweeping, no pinching.

Finite size scaling due to lowest mode $q_1 = \pi/L$.

At or around criticality



System size $L = 128$, $L = 256$, $L = 512$, $L = 1024$.

The Manna Model is **at** criticality: No hovering, no sweeping, no pinching.

Finite size scaling due to lowest mode $q_1 = \pi/L$.

Suitable observables

The **substrate** is a good place to look for **self-organisation**.

- The particle density adjusts, but its value is not universal (value to be compared to the *same* system).
- Correlations in the substrate may be absent or very weak. They occur *to counter* scaling in the dynamics.

The substrate is a bad place to look for criticality.

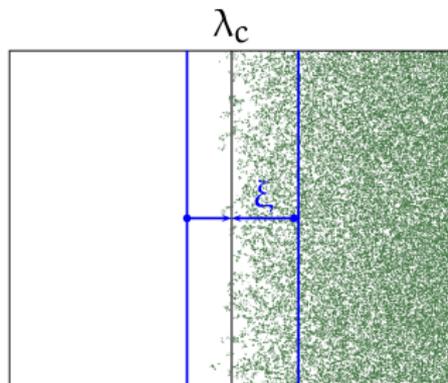
Suitable observables

The **activity** is a good place to look for **scaling** (integrated activity: avalanche metrics).

- Finite size scaling.
- Change of resolution.
- Thresholding? (may introduce spurious crossover)
- Block scaling (conditional to activity).
- Scaling should be compared to null models (is it just white noise?).
- Exponents are (supposedly) universal.
- Moment ratios are (supposedly) universal.

Block scaling

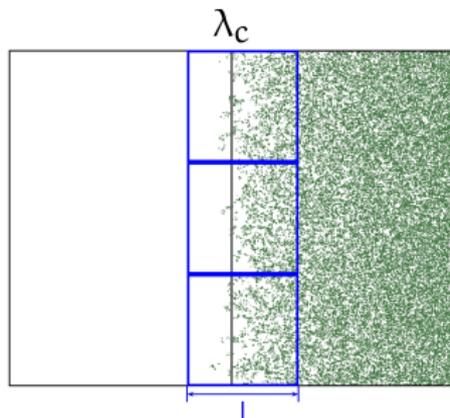
... is a form of subsampling.



- Change of system size may be impossible (how about resolution, threshold — dangerous!).
- **Block finite size scaling:**
Measure densities and fluctuations in varying box sizes.

Block scaling

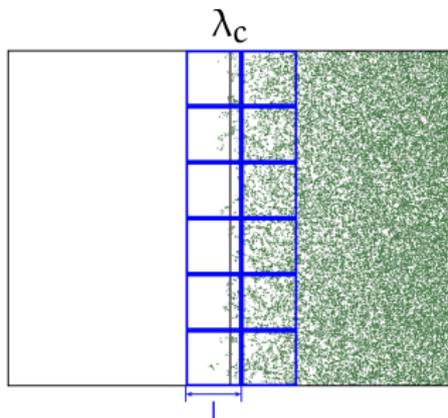
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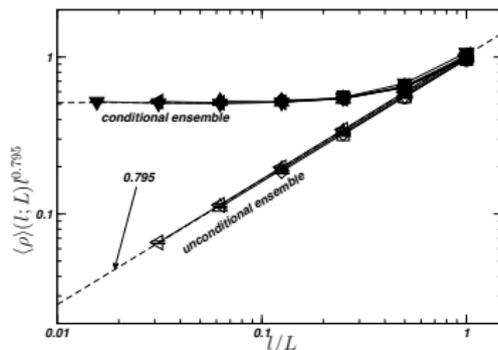
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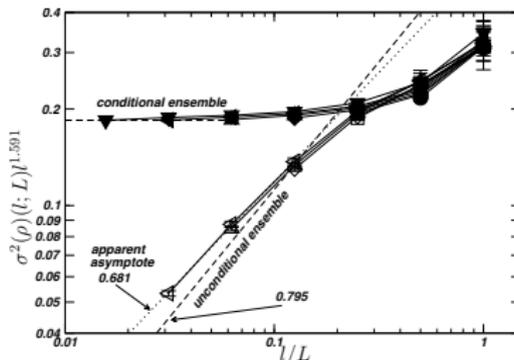


Contact process, Pruessner 2008, Fig. 1

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Contact process, Pruessner 2008, Fig. 2

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Summary

- A solid definition of SOC is hard to come by.
- I propose: Scaling (non-trivial, spatio-temporal, finite size), self-organisation to a critical point, intermittency, non-linear interaction.
- Henrik Jensen: SDIDT (slowly driven, interaction dominated, threshold systems).
- Field theory: Truly *at* the critical point.
- Observables: Scaling to be found in the activity, not the substrate.
- Block scaling?

THANKS!

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