Identifying Self-Organised Criticality in nature

A guide by the confused

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Bern (ISSI meeting), September 2013

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Outline

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What is SOC?

As far as general use goes, what does SOC normally refer to?

Two extremes:

- Anything where "critical behaviour" is observed without tuning of a parameter.
- Anything avalanching.

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What is SOC?

Critical behaviour without tuning?

Typical criticism:

- Is the Ising Model at $T = T_c$ SOC?
- Is percolation SOC ($p_c = 1/2$ for square, bond and triangular, site)?
- Is a fair random walker SOC?
- Is a fair branching process SOC?
- Is turbulence SOC?

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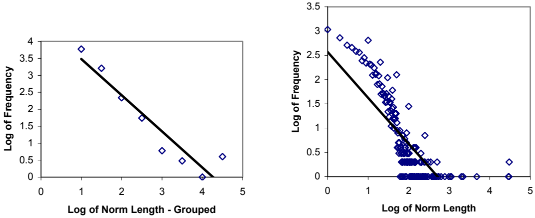
What is SOC?

A footnote on turbulence

- Scaling largely a matter of dimensional analysis (trivial?)
- Separation of time scales in “output” rather than driving (Grinstein, 1995)
- Flow of energy to *smaller and smaller* length scales.
- Definition of avalanches only via explicit thresholding (not those of the dynamics)

What is SOC?

Anything avalanching?



Hoffmann, 2005, Figs. 9 and 10

- Wars (Roberts & Turcotte, 1998)
- Pop charts (Bentley & Maschner, 1999)
- Urban Development (Batty & Xie, 1999)
- Hospital waiting times (Smethurst & Williams, 2001)
- Avalanches of social norms (Hoffmann, 2005)

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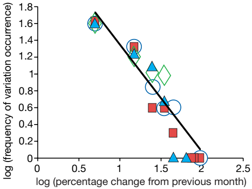

Smethurst & Williams, 2001, Fig. 1

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What is SOC?

Anything with a power law ?

- Gravity, $F \propto r^{-2}$
- Hospital waiting times (Smethurst & Williams, 2001)
- Percolation

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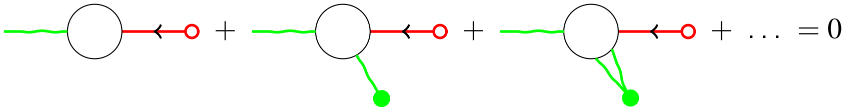
SOC

1. Non-trivial Scaling (finite size scaling — no control parameter)
2. Spatio-temporal correlations
3. Apparent self-tuning (underlying 2nd order phase transition?)
4. Separation of time scales
5. Avalanching (intermittency)
6. [nonlinear (thresholds) interaction] (supposedly required by 1)

SOC: Non-trivial scale invariance (spatio-temporal correlations!) in avalanching (intermittent) systems as known from ordinary critical phenomena, but with internal, self-organised rather than external tuning of a control parameter (to a non-trivial value).

The (field) theory of SOC

Stationarity is equivalent to self-organisation to critical point.
Stationarity (lack of additional net deposition):



- Vanishing deposition at stationarity means that the diagrams in the bracket vanish .
- Requires adjustment of substrate .
- Independent of driving .

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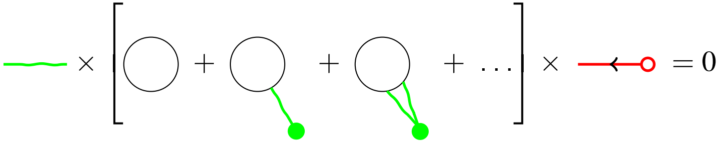

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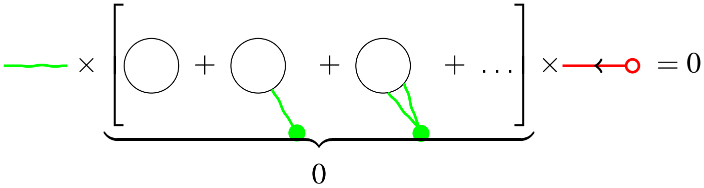

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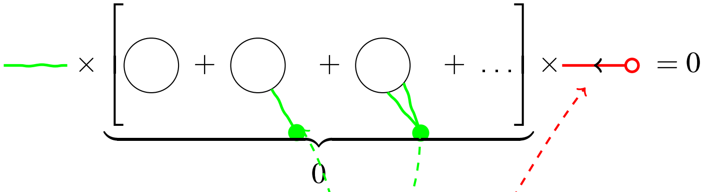

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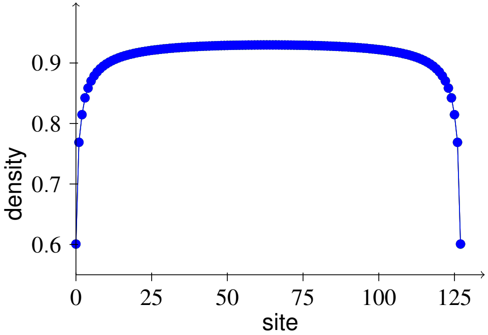
At or around criticality



Driving uniformly, at site 1, at site 0.

The Manna Model is at criticality: No hovering, no sweeping, no pinching.
Finite size scaling due to lowest mode $q_1 = \pi/L$.

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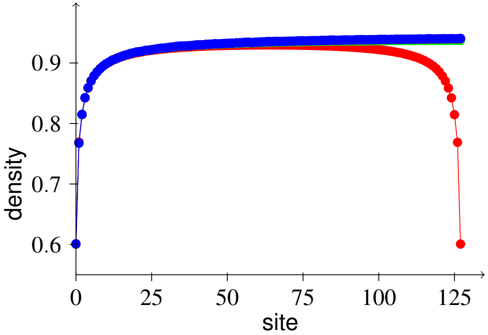
At or around criticality



System size $L = 128$, $L = 256$, $L = 512$, $L = 1024$.

The Manna Model is at criticality: No hovering, no sweeping, no pinching.
Finite size scaling due to lowest mode $q_1 = \pi/L$.

Suitable observables

The substrate is a good place to look for self-organisation.

- The particle density adjusts, but its value is not universal (value to be compared to the *same* system).
- Correlations in the substrate may be absent or very weak. The occur *to counter* scaling in the dynamics.

The substrate is a bad place to look for criticality.

Suitable observables

The activity is a good place to look for scaling (integrated activity: avalanche metrics).

- Finite size scaling.
- Change of resolution.
- Thresholding? (may introduce spurious crossover)
- Block scaling (conditional to activity).
- Scaling should be compared to null models (is it just white noise?).
- Exponents are (supposedly) universal.
- Moment ratios are (supposedly) universal.

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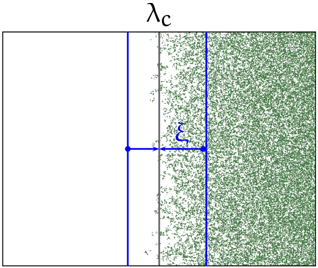
Block scaling

... is a form of subsampling.



- Change of system size may impossible (how about resolution, threshold — dangerous!).
- Block finite size scaling:
  Measure densities and fluctuations in varying box sizes.

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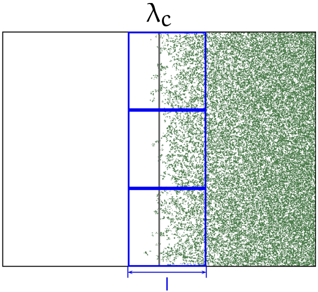

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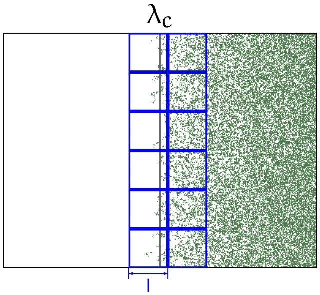

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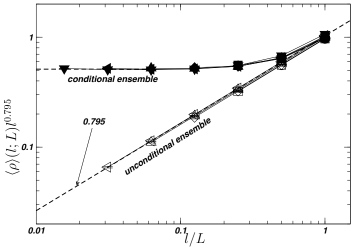

Contact process, Pruessner 2008, Fig. 1

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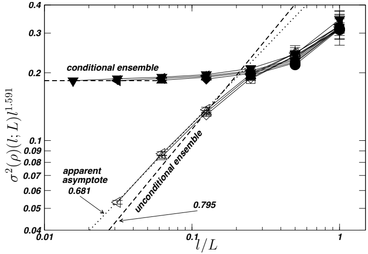

Contact process, Pruessner 2008, Fig. 2

- Change of system size may impossible (how about resolution, threshold — dangerous!).
- Block finite size scaling:
  Measure densities and fluctuations in varying box sizes.

Summary

- A solid definition of SOC is hard to come by.
- I propose: Scaling (non-trivial, spatio-temporal, finite size), self-organisation to a critical point, intermittency, non-linear interaction.
- Henrik Jensen: SDIDT (slowly driven, interaction dominated, threshold systems).
- Field theory: Truly *at* the critical point.
- Observables: Scaling to be found in the activity, not the substrate.
- Block scaling?

THANKS!

Supported by the ISSI and EPSRC platform grant EP/I019111/1.

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